Sample Chapter

The History of the Theory of Structure

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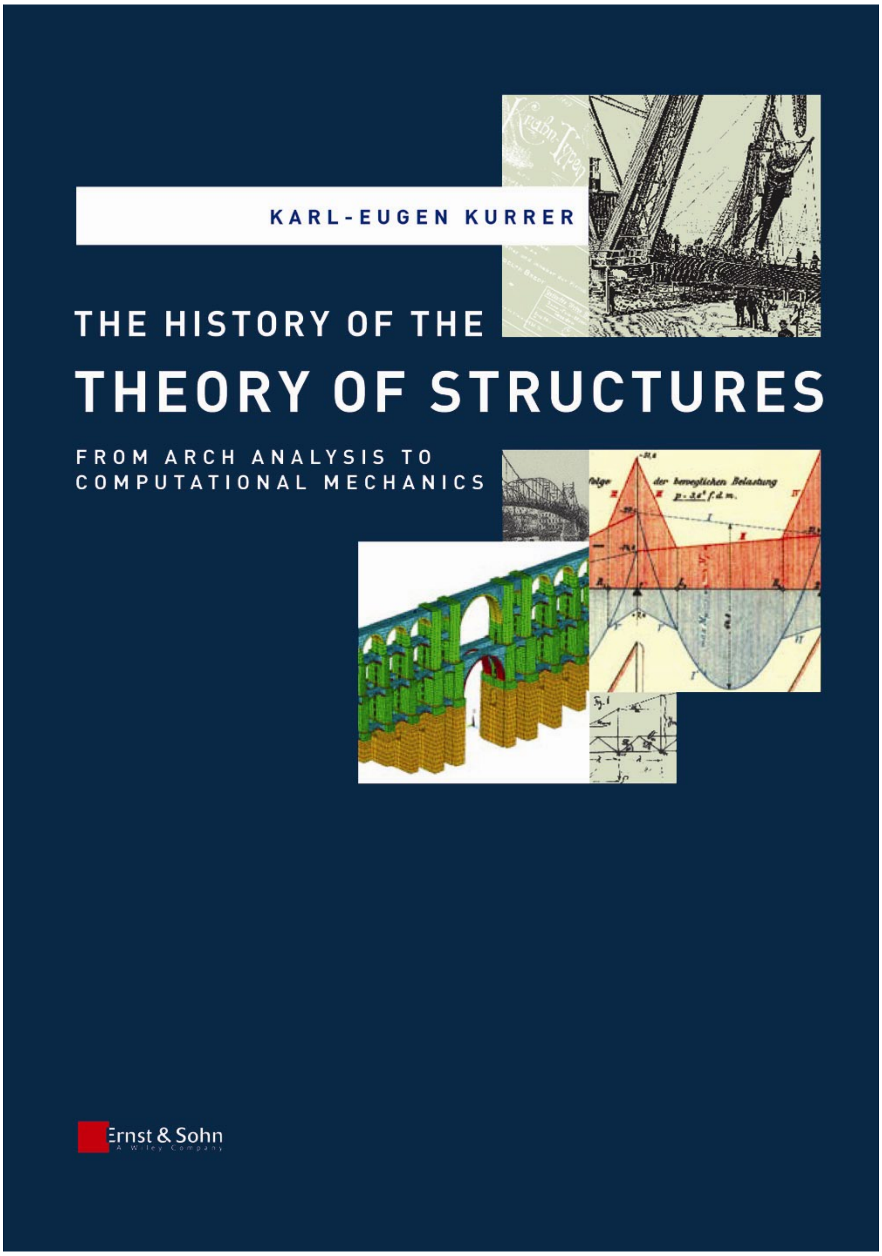

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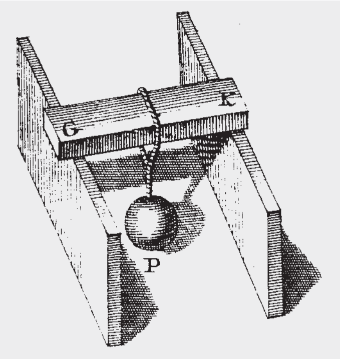
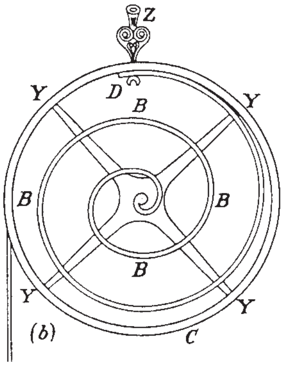
CHAPTER 5



The beginnings of a theory of structures



Like all roads in the Roman Empire led to Rome, so we can trace strength of materials back to Galileo's *Discorsi* of 1638. Historically, statics and strength of materials had to be found in theory of structures. The author's interest in Galileo stems not only from Bertolt Brecht's famous play *The Life of Galileo*, but also from an in-depth study of the philosophical history of mechanics writings of Pierre Duhem, Eduard Jan Dijksterhuis, Michael Wolff, Gideon Freudenthal and Wolfgang Lefèvre. In 1980, while still a student, the author purchased a copy of Franz Joseph Ritter von Gerstner's scientific life's work, the three-volume *Handbuch der Mechanik* (manual of mechanics) with its magnificent copperplate engravings. An intensive study of this publication and the work of Johann Albert Eytelwein led to the author's view that these two personalities rounded off the preparatory period of structural theory, but in the end were unable to formulate the programme of structural theory. That was to be left to Navier, who in 1826 fused together statics and strength of materials to form theory of structures. And that's where the history of theory of structures, in its narrower sense, really begins. The results have been published by the author in the yearbook *Humanismus und Technik* edited by Prof. Rudolf Trostel.

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The process of the scientific revolution in the 17th century was characterised by the fact that the emergence of the modern natural sciences shaped by Galileo, Descartes and Newton resulted in the natural sciences leaving the production sphere on the social scale and progressing to a separate sphere of social activities.

The natural sciences approach to analysing simple technical artefacts is evident in Galileo's important work *Dialogue Concerning Two New Sciences* (1638) due to the fact that he embraces both nature and engineering mathematically, i.e. describes them as a world of idealised objects. Galileo ignores disturbing influences in the formulation of the law of falling bodies and realises concrete technical artefacts as idealised theoretical models (tensile test, bending failure problem). Galileo's questioning of the difference of geometrical and static similarity for objects in nature and engineering forms the very heart of his strength of materials investigations; its origin lies in his idealisation of objective reality through mathematics, which for him is essentially another theory of proportions.

Whereas mechanics before Galileo and Newton was able to express theoretically simple problems of engineering, like the five simple machines (lever, wedge, screw, pulley, wheel on axle), only in isolated instances, the scientific system of theoretical mechanics evolving dynamically in the 18th century was now able to express all those technical artefacts whose physical behaviour is determined principally by the laws of mechanics. The contradiction between the scientific understanding and complexity of the technical artefacts primarily analysed by mechanics (beam, arch, earth pressure on retaining walls, etc.) is evident not only in all the scientific works of the most important mathematicians and mechanics theorists of the 18th century, but also in the gulf between those and the advocates of the practical arts – the hands-on mechanics, the mill-builders, the mining machinery builders, the instrument-makers and the engineers. In the scientific system of mechanics, theoretical mechanics dominated until its social impact was directed at the creation of a comprehensive scientific conception of the world for the rising middle classes.

It was not until the Industrial Revolution took hold in Great Britain around 1760 – i.e. the transition from workshop to factory – did the conscious link between natural science knowledge activities and engineering practice become the necessary historical development condition for the productive potency of society. As criticism of the static theory of proportion in strength of materials investigations became loud during this period, so a scientific basis for this branch of knowledge began to emerge, simultaneously with the mechanisation of building, finally crystallising in the first three decades of the 19th century as the fusion of these two processes in the form of a theory of strength of materials and a theory of structures. What was now needed was neither a geometrical theory of proportion, which we find in the master-builders of the Renaissance, nor a static proportions theory, as Gerstner was still using to size his beams, but rather the unity of strength test and theoretical modelling of loadbearing structures in construction engineering. The discipline-formation period of structural theory began with the formulation of Navier's theory of structures programme in 1826.

5.1 What is the theory of strength of materials?

The resistance of a solid body to its mechanical separation by external mechanical-physical actions is a principal property of solid bodies. We call this property "strength".

For instance, in a tensile test a steel wire of cross-section $A = 1\ \text{mm}^2$ opposes the external tensile force F by means of an equal internal tensile resistance (Figs 5-1a and 5-1b). If the applied tensile force increases beyond the yield point F_{yield} until failure F_B of the steel wire, the internal tensile resistance F_B is overcome. We then say that the tensile strength of the steel wire has been reached (Figs 5-1c and 5-1d). Fig. 5-1d illustrates the associated stress-strain diagram for the steel wire (mild steel grade S 235 to Eurocode EC 3, i. e. with a minimum tensile strength of $360\ \text{N/mm}^2$). Force F increases linearly with the deformation Δl until it reaches the yield point at $F_{yield} = 240$ N. In the yield zone, the steel wire continues to extend while the force F_{yield} remains constant at 240 N. Afterwards, it enters the plastic phase until it fails (ruptures) at $F_B = 360$ N. For calculations according to plastic hinge theory, the stress-strain diagram according to Fig. 5-1d is simplified to Fig. 5-1e: we then speak of ideal elastic (range: $0 \leq F \leq F_{yield} = 240$ N) and ideal plastic (range: $F = F_{yield} = 240$ N) material behaviour. Tensile strength is today described in terms of stress, i. e. in the magnitude of a force per unit surface area (e. g. N/mm^2). Consequently, the tensile test can reveal the internal tensile resistance at every stage, e. g. by way of a calibrated force scale (Fig. 5-1d).

This rendition of the invisible, this exposure of the internal tensile resistance, is particularly evident at the moment of failure, when the specimen splits into two parts (Fig. 5-1c). The method of sections of mechanics (see Fig. 5-1b) is in its simplest form the imaginary emulation of the rupture process in the tensile test. If the engineer wishes to calculate the internal forces of the members of a truss at a joint, he cuts them apart at the joints in a thought experiment and thus "exposes" the internal member forces at the joint under consideration. He determines their magni-

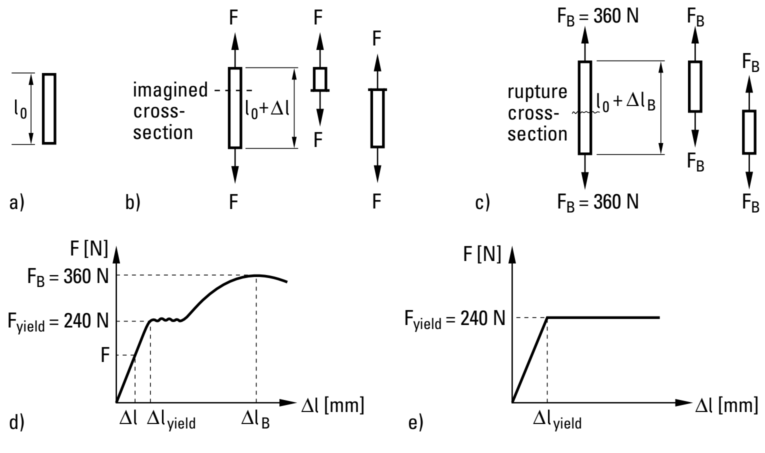


FIGURE 5-1
Schematic diagram of a tensile test with associated stress-strain diagram for mild steel grade S235: a) specimen of length l_0 and cross-sectional area $A = 1\ \text{mm}^2$, b) stress-strain condition in the elastic range of stress, and c) at failure; d) stress-strain diagram, e) simplified stress-strain diagram for ideal elastic and ideal plastic material behaviour.

tude and direction (tension or compression) in such a way that the joint remains in a state of equilibrium. Without the thought experiments of Leonardo da Vinci (1452–1519) concerning the tensile strength of wires, without those of Galileo Galilei (1564–1642) concerning the tensile strength of copper wires, ropes and marble pillars, without the numerous tensile tests of the natural scientists and engineers of the 18th and early 19th century on the one hand and the kinematic analysis of simple machines (especially the pulley) on the other, the method of sections, which Lagrange's work *Méchanique analytique* (1788) had already explained in theory, would not have become the fundamental method of mechanics in the 19th century. The tensile test therefore marks the beginnings of a theory of strength of materials, in the historical and the logical sense. Although modern strength of materials theories take account of bending, compression, shear and torsional strength as well as tensile strength, Galileo's tensile tests in the form of thought experiments formed the germ cell of a theory of strength of materials. Only very few textbooks on strength of materials do not begin by describing the tensile test (Fig. 5-1); not only the method of sections, but also the codified relationship between the two principal mechanical variables can be verified empirically in a particularly simple and convincing way by measuring the tensile force F and the elongation Δl. By contrast, identifying the effects due to bending, shear and torsion are much more complicated. After the formulation of the bending problem by Galileo in 1638, almost two centuries passed before Navier's practical bending theory (1826) enabled engineers to understand reliably the bending strength of beam-type construction elements. Fig. 5-2 (Fig. 1 to Fig. 9) gives the reader an impression of the strength problems scientists were already analysing in the early 18th century. Besides the loading due to tension (Fig. 1 and Fig. 8), we have beams subjected to bending, such as the cantilever beam (Fig. 2) plus the simply supported and the fixed-end beams (Figs 3 and 4 respectively) on two supports; there is also a column fixed at its base loaded in compression (Fig. 5). Even the strength of prismatic bodies to resist compression – at first sight just as easy to analyse as the strength to resist tension – turned out to be a difficult mechanical problem in the case of slender struts (buckling strength), which was not solved satisfactorily for steelwork much before the end of the 19th century. The tensile test and its mechanical interpretation is therefore at the historico-logical heart of the emerging theory of strength of materials.

According to Dimitrov, strength of materials is the foundation of all engineering sciences. It aims to "provide an adequate factor of safety against the unserviceability of the construction" [Dimitrov, 1971, p. 237]. That historian of civil engineering, Hans Straub, regards strength of materials as "the branch of applied mechanics that supplies the basis for design theory", with the task of "specifying which external forces a solid body … may resist" [Straub, 1992, p. 389]. Whereas Straub places elastic theory firmly in the strength of materials camp, Istvan Szabó distinguishes the aforementioned scientific disciplines according to their objectives: the aim

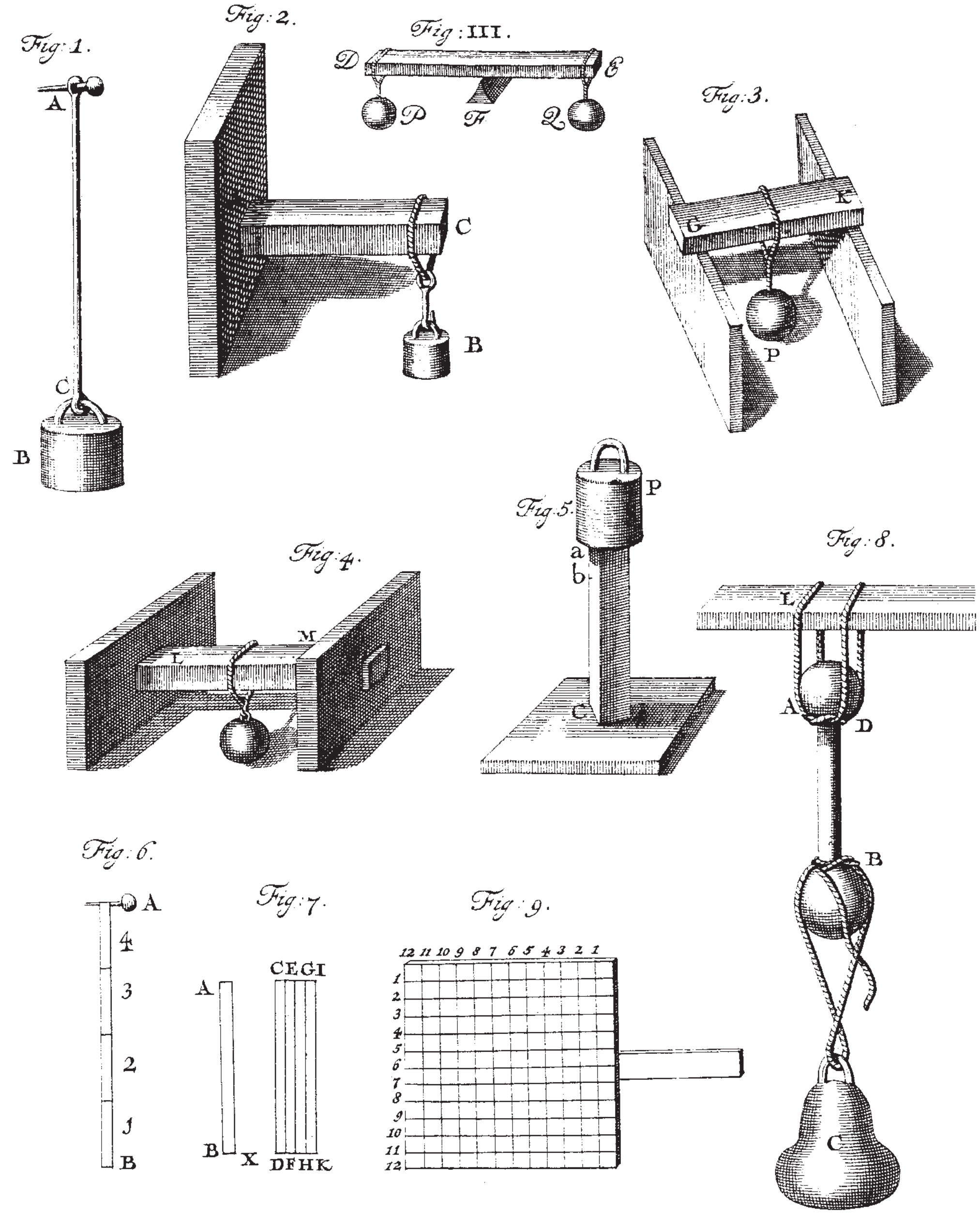


FIGURE 5-2
Some strength problems of the early 18th century

of elastic theory is to determine the deformation or displacement condition for a body with a given form subjected to an applied load, "whereas strength of materials regards the load on a body as known when the (internal) stresses, for which we prescribe permissible limits depending on the material, are calculated in addition to the displacement" [Szabó, 1984, p. 84]. On the other hand, the "Old Master" of applied mechanics, August Föppl, places the examination of the displacement condition and the associated stress condition in the focal point of the object of strength of materials, which can therefore be understood as the "mechanics of internal forces" [Föppl, 1919, p. 3]. "Strength of materials," write Herbert Mang and Günter Hofstetter, "is a customary abbreviation of the engineering science discipline of the applied mechanics of deformable solid bodies. The mechanics of deformable bodies is a discipline belonging to continuum mechanics … The main task of strength of materials is to calculate stresses and deformations, primarily in engineering constructions" [Mang & Hofstetter, 2004, p. 1].

The purpose of strength of materials is to portray quantitatively and qualitatively the resistance of solid bodies to their mechanical separation by mechanical-physical actions with the help of experiments and theoretical models, and to prepare these in the form of an engineering science knowledge system in such a way that they can be used as a resource in engineering activities. Consequently, strength of materials is based, on the one hand, on the practical experiences of materials testing plus the science of building and materials, but, on the other, also the theoretical models of the applied mechanics of deformable solid bodies.

5.2 On the state of development of structural design and strength of materials in the Renaissance

When we look at the domes of the late ancients, the delicate loadbearing systems of the Gothic period and the long-span masonry arch bridges of the Renaissance, it is not unusual to ask the question of whether their builders did not perhaps have some knowledge of theory of structures on which to base their bold designs. As A. Hertwig, that aficionado of the history of building and building theory, wrote in 1934: "If we study the Hagia Sophia [in Istanbul – the author] built in 537 AD or the Pantheon [built in 27 BC in Rome – the author] from a structural viewpoint, then we discover a cautious exploitation of the various material strengths which could not have been achieved just by using the simple rules for the design of individual elements. The builder of the Hagia Sophia, Anthemios, was revered by his contemporaries as a mathematician and mechanical engineer. That can mean nothing other than he prepared structural calculations for his structures. The knowledge of mechanics at that time would have been wholly adequate for this purpose. For with the help of Archimedes' (287 – 212 BC) principle concerning the equilibrium of forces in a lever system it is certainly possible to investigate the limit states of equilibrium in masonry arches and pillars by considering all the parts as rigid bodies" [Hertwig, 1934/2, p. 90]. On the other hand, based on careful history of building and history of science studies, R. J. Mainstone comes to the following conclusion: "No quantitative application of statical theory is recorded before the time of Wren" [Mainstone, 1968, p. 306]. The architect and engineer Christopher Wren (1632 – 1723), friend of Isaac Newton (1643 – 1727), was born in the year that Galileo's *Dialogue* was published.

However, were not individual principles of structural theory and strength of materials recognised and integrated in qualitative form into the engineering knowledge surrounding buildings and machines? In a paper, S. Fleckner presents his comparative structural investigations of the great Gothic cathedrals and formulates the thesis that their characteristic buttresses are dimensioned on the basis of structural calculations

[Fleckner, 2003, p. 13]; however, he considers only the circumstantial evidence without naming any positive contemporary sources. Nonetheless, the above question cannot be rejected for the period of the Renaissance. In the drawings of Leonardo da Vinci (1452 – 1519), it is possible to find numerous examples of the principles of structural theory and strength of materials which eclipse the resources of outstanding contributors from the Hellenistic phase of ancient science such as Archimedes, Heron (around 150 BC) and Ktesibios (around 250 BC), and are closely linked with Leonardo's engineering thinking. Also famed as the painter of "Mona Lisa" and "The Last Supper", this engineer provides the first written evidence of a strength experiment (Fig. 5-3) in his notebooks from around 1500 well known to historians of culture, science and technology under the title of *Codex Atlanticus*.

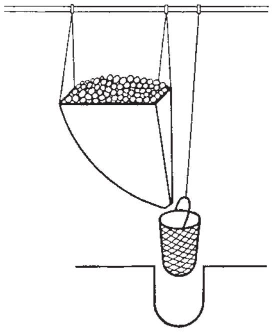


FIGURE 5-3
Leonardo da Vinci's test setup for determining the tensile strength of a wire

Leonardo describes his test thus: "Experiment concerning the load that wires of different length can carry. Perform the experiment to find out how much weight an iron wire can hold. You should proceed as follows in this experiment: Hang an iron wire 2 cubits long [1 Milanese cubit = approx. 600 mm – the author] from a place that holds it firmly. Then hang a basket or similar on the wire in which there is a small hole with which to fill to fill a basketful with fine sand from a funnel. When the iron wire can no longer carry the load, it breaks. … Note the magnitude of the weight as the wire broke and also note at which point the wire breaks. Perform the experiment again and again in order to discover whether the wire always breaks at the same point. Then halve the wire and observe whether it can now carry more weight. Then shorten it to one-quarter of the initial length and gradually, using different lengths, you will discover the weight and the place at which the wire always breaks. You can carry out this test with any material – wood, stone, etc. Set up a general rule for each material" [Krankenhagen & Laube, 1983, p. 31]. When we read the description of the experiment, it is easy to gain the impression that Leonardo had not recognised the cause of the different tensile failure forces for the different specimen lengths. However, this impression is totally refuted when we study the relevant passages in his notebooks rediscovered in 1965 in the Madrid National Library, which is why they are called *Codex Madrid I and II*. Leonardo recognised firstly that "it is possible for a vertical, suspended rope to break due to its own weight", and secondly that "the rope breaks where it has to carry the greatest weight, i. e. at the top where it is connected to its support" [Leonardo da Vinci, 1974/2, p. 204]. Leonardo had therefore not only anticipated qualitatively the notion of the breaking length, but had also indirectly answered the question regarding the rupture cross-section in the wire strength test he so carefully described: the wire in the test, too, must theoretically break at the point of its suspension because this is where the total self-weight of the iron wire of length l is added to the weight of the basket and the sand (Fig. 5-3). If we reduce the length of the iron wire in the test, the shortened wire can carry an additional weight of sand corresponding to the weight of the "missing" length of wire. And vice versa: lengthening the wire in the test results in an equivalent reduction in the weight of sand that can be carried at the moment of failure; when we reach the breaking length, the iron wire parts under its own weight. But the *Codex Madrid*, with their sensational findings for the history of science and technology, also contain statics-constructional knowledge which, had it been available in the Renaissance, would have helped Galileo's strength experiments enormously.

Probably inspired by his designs for a giant crossbow, Leonardo analysed the relationship between external load and deformation on pre-bent and straight elastic bars. The intuitive knowledge of the codified relationship between pretensioning force and deformation is a prerequisite for giant crossbows that cannot be pretensioned manually because the deformation energy stored in the elastic bar after releasing the pretension is converted almost exclusively into the kinetic energy of the projectile; as the load increases, so the elastic deformation also increases – and hence the stored deformation energy. Fig. 5-4a shows an elastic bar loaded in the middle successively with the weights G, $2G$, $3G$, $4G$ and $5G$. Leonardo now wanted to know "the curvature of the [elastic] bar, i. e. by how much it differs, larger or smaller, from the other weights. I believe that the test with twice the weight in each case will show that the curvatures behave similarly" [Leonardo da Vinci, 1974/1, p. 364].

Although the notion of the curvature κ of a curve as the inverse of the radius of curvature was not expressed mathematically until the turn of the 18th century in the investigations into elastic lines, Leonardo had recognised the material law for specific elastic lines: from the proportionality between external load G and internal bending moment M on the one hand and the proportionality between M and the curvature κ of the elastic line on the other, it follows that there should be proportionality between external load G and curvature κ as asserted by Leonardo. Contrasting with this is Leonardo's mistaken assertion that the bending deflection f of two beams subjected to the same central load G is identical when the longer beam is four times the length and the cross-section twice as wide and twice as deep (Fig. 5-4b). If the bending deflection of the short beam

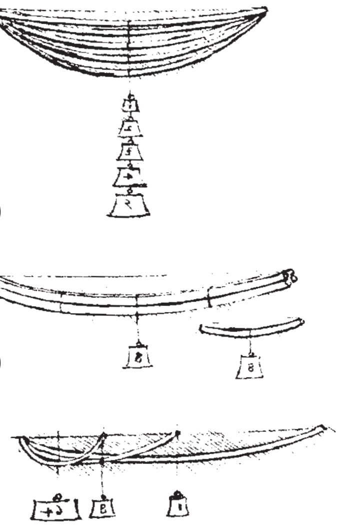
$$f = \frac{G \cdot l^3}{48 \cdot E \cdot I} \tag{5-1}$$

where span = l, elastic modulus of the material = E and second moment of area $I = b \cdot h^3/12$ (b = width of cross-section, h = depth of cross-section), then if instead of l, $l' = 4\,l$ and instead of I, $I' = [2 \cdot b \cdot (2 \cdot h)^3]/12$ are used for the longer beam,

$$f' = \frac{G \cdot l^3}{48 \cdot E \cdot I} = 4 \cdot f \tag{5-2}$$

i. e. the longer beam exhibits four times the bending deflection of the shorter one.

Leonardo's assertion would be right if the bending deflection increased by only the square of the span, i. e. the bending line was a quadratic and not a cubic parabola. However, Leonardo's answer to the following problem is correct (Fig. 5-4c): "I shall take three bars of the same thickness, one of which is twice as long as the others. And each shall be subjected to



a)

b)

c)

FIGURE 5-4
a) Deformations of elastic bars of constant cross-section and equal length subjected to various point loads at the middle of the bar, b) deformations of elastic bars of different cross-sections and lengths subjected to an identical point load in the middle of the bar, c) deformations of elastic bars of constant cross-section but different lengths subjected to various point loads in the middle of the bar.

a load in the middle such that the curvatures exhibit the same deflection" [Leonardo da Vinci, 1974/1, p. 364]. According to Leonardo, the proportion of the loads to the spans must be $G : 8G : 64G = 1 : 0.5 : 0.25$.

The bending deflection f of the longest beam of span l under load G can be calculated using eq. 5-1. The load must equal $8G$ to produce the same bending deflection f when the span is halved, which is easy to establish by entering the values into eq. 5-1. If we use the span $l/4$ instead of l in eq. 5-1, the load must increase to 64 times the value of the original load for the beam of span l in order to achieve the same bending deflection.

As the treatment of construction engineering issues in Leonardo's notebooks is secondary to his qualitative analysis of machine elements and does not form a coherent system of construction engineering knowledge, the following is merely a summary of some potential solutions that today would fall within the remit of theory of structures and strength of materials:

– Leonardo recognised the principle of resolving a force into two components, but without determining those components quantitatively [Wagner & Egermann, 1987, p. 179].
– He used the notion of the static moment (force multiplied by lever arm) for the first time on inclined forces [Straub, 1992, p. 91].
– Leonardo anticipated the linear strain distribution (beam cross-sections remain plane) of elastic beams with a rectangular cross-section as assumed by Jakob Bernoulli in 1694 [Kurrer, 1985/1, p. 3].
– In the static analysis of masonry arches, he developed a wedge theory in which he satisfies the moment equilibrium of each voussoir but neglects the displacement (translation) equilibrium despite being aware of the parallelogram of forces; further, he specified possible collapse mechanisms of asymmetrically and symmetrically loaded masonry arches [Zammattio, 1974, p. 210].
– Leonardo sketched out a method in which the horizontal thrust of diverse arch forms and couple roofs could be analysed experimentally and, as a result, how thick the abutments would have to be [Zammattio, 1974, p. 211].
– The load-carrying capacities of concentrically loaded fixed-end columns behave inversely proportional to their height; whereas with a concentrically loaded column there are no flexural deformations, the fibres of eccentrically loaded columns are extended on the side opposite to the load but compressed on the loaded side.

In fact, the French science historian Pierre Duhem claims that Leonardo's pupil Francesco Melzi passed on the notebooks, either as originals or copies, to Cardano, Benedetti, Guidobaldo del Monte and Bernardino Baldi, and via these scientists could have influenced Galileo [Straub, 1992, p. 94]. But we can also assume that Leonardo's "chaotic collection of notes in which ingenious ideas and everyday extracts from acknowledged works are intermingled" [Dijksterhuis, 1956, p. 283] may not have been able to accelerate scientific and engineering progress precisely because of the lack of an orderly presentation and their coincidental distribution. Mind you, these two remarks were expressed prior to the rediscovery of the *Codex Madrid*! The fact that at least some of the aforementioned structural theory findings of Leonardo were not taken up in the subsequent two centuries is illustrated by using Philippe de la Hire's masonry arch theory (first published in 1695) as an example. Here, in contrast to the masonry arch theory of Leonardo, the translation equilibrium of the frictionless-jointed voussiors of a semicircular masonry arch is assumed, i. e. the theory requires the magnitude of the external forces required to keep the – expressed in modern terms – system of hinges (with multiple kinematic indeterminacy) just in (unstable) equilibrium. Therefore, Leonardo's (building) design theory findings remain as erratic uplands in the emerging new scientific landscape between which paths of scientific knowledge pass. The grandiose individual (building) design theory findings of Leonardo da Vinci were not recognised by subsequent generations of scientists and engineers in the age of the scientific revolution.

In 1586 the Dutch mathematician Simon Stevin (1548 – 1620) carried out the "wreath of spheres" experiment to prove the law of the inclined plane in his work *Beghinselen der Weeghconst* (The Elements of the Art of Weighing). As professor of mathematics and senior waterways engineer in the Netherlands, Stevin had noticed the difference between theoretical and applied mechanics but had underestimated its extent: "Once he had determined theoretically the ideal equilibrium condition between force and load in a tool, then he was of the opinion that a very minor increase in the force could now set the load in motion. Nonetheless, owing to the many applications that made use of his theoretical investigations (in weighing and lever tools, in windmills, the horse's bridle and in military science), he advanced both *Weeghconst* (the art of weighing = theoretical statics) and *Weghattet* (the practice of weighing = practical statics)." [Dijksterhuis, 1956, p. 365]

The relationship with engineering, as we experience in Stevin's scientific work on statics, remained more of an exception in the late Renaissance. And conversely, we cannot speak of progress in engineering steered by theory. The technical problems of construction, mining and mechanical engineering were often too complex to be adequately catered for scientifically in the laws of nature effective in engineering entities and methods – indeed, even to be anticipated theoretically. And that is why the Fleisch Bridge in Nuremberg, completed in 1598, with its spectacular rise-to-span ratio of 4 m to 27 m ($f : l = 1 : 6.75$) and highly complex subsoil conditions cannot possibly be the result of an analysis based on a structural theory [Falter et al., 2001]. Assuming elastostatic behaviour in the uncracked condition, Karl Krauß discovered in his re-analysis that the Fleisch Bridge should exhibit cracks on the underside of the crown of the arch. "Despite several detailed investigations of the masonry, I found neither cracks nor signs of repairs" [Krauß, 1985, p. 220]. During his examination of the archive material, Krauß noticed the short construction period (about two months to construct the masonry arch of the bridge) and he turned his attention to the problem of the lime mortar, which would have been still soft upon completion. Another structural check assuming plastic deformation of the fresh mortar in the joints revealed that the crack zone at the crown calculated previously had now disappeared. "Sensitised by this probing of the mortar's influence, I read Alberti's work [*On the Art of Building in Ten Books*, Florence, 1485 – the author] again and discovered in chapter 14 of the third book a method of striking the centering of masonry arches, the significance of which had not been realised: 'And apart from that it is good in centred masonry arches to relieve the support a little (forthwith), where they are completed by the uppermost voussoirs (by which the centering is supported so to speak) so that the freshly built voussoirs do not float between their bed and the lime mortar, but rather assume a balanced, steady position of complete equilibrium with one another. If this happens during the drying, however, the masonry would not be compressed together and hold, as is necessary, but would leave cracks upon settling. The work should therefore be carried out as follows: the centering should not be simply removed, but instead gradually loosened from day to day so that the fresh masonry does not follow if you take the centering away prematurely. After a few days, however, depending on the size of the arch, loosen the centering a little more. And then continue until the voussoirs fit to the arch and to one another and the masonry stiffens' " [Krauß, 1985, p. 220]. The structural re-analysis by Krauß taking into account the plastic deformation of the joints in the masonry arch and the passage from Alberti's work quoted by him is an impressive demonstration of how the builders of the Fleisch Bridge were able to "influence the static behaviour of the masonry by controlling the progress of the work" [Krauß, 1985, p. 221] even without structural calculations. In her dissertation *Die Fleischbrücke in Nürnberg 1596 – 98* (2005), which is based on a critical evaluation of the extensive archive material available, Christiane Kaiser arrives at the conclusion that merely qualitative statics-constructional deliberations were represented graphically in the designs for this structure [Kaiser, 2005].

Until well into the discipline-formation period of structural theory in the 19th century, the relevance and practicability of the experience of builders accumulated at the interface between the construction process, structural design and static behaviour remained far superior to that gained in theoretical trials. It was not until the conclusion of the consolidation phase of structural theory (in the 1950s), as civil and structural engineers had at their disposal the knowledge of materials in a scientific form and even industrialised methods of building had access to a scientific footing (and therefore the interaction between progress on site and statics-constructional scientific attained relevance), that we can speak for the first time of the supremacy of the experience at that interface being overtaken by scientifically founded knowledge.

5.3 Galileo's *Dialogue*

Long chapters have often been devoted to the *Dialogue* (Fig. 5-5) and its embedment in the process of forming the new natural sciences in monographic summaries on the history of the natural sciences. With only a few exceptions, these contributions to the history of science concentrate their analysis on the dynamic of Galileo as one of the "two new sciences" (Galileo), whereas the individual problems of Galileo's strength tests are mentioned only in passing in the bibliography on the history of technology and engineering sciences, like in M. Rühlmann [Rühlmann, 1885], S. P. Timoshenko [Timoshenko, 1953], F. Klemm [Klemm, 1979], T. Hänseroth [Hänseroth, 1980], E. Werner [Werner, 1980], K. Mauersberger [Mauersberger, 1983], Hänseroth and Mauersberger [Hänseroth & Mauersberger, 1987], H. Straub [Straub, 1992], and P. D. Napolitani [Napolitani, 1995]. Although historians of engineering sciences have already achieved significant successes in unravelling the early evolutionary stages of structural mechanics [Hänseroth, 1980] and applied mechanics [Mauersberger, 1983], a detailed analysis of Galileo's strength experiments still represents a gap in the research.

Galileo's *Dialogue* is a discussion between Salviati (the voice of Galileo), Sagredo (an intelligent layman) and Simplicio (representing Aristotelian philosophy) about two new sciences, and unfolds over a period of six days:

First day: Tensile strength of marble columns, ropes and copper wires; reference to cohesion; mathematical considerations; free fall in a vacuum and in a medium; pendulum motion, etc.

Second day: Consideration of the ultimate strength of beams with different forms, loadings and support conditions under similarity mechanics aspects.

Third day: Law of falling bodies

Fourth day: The motion of projectiles

Fifth day: Theory of proportion

Sixth day: Force of percussion

Only the first two days are of immediate interest for the history of applied mechanics or the theory of strength of materials.

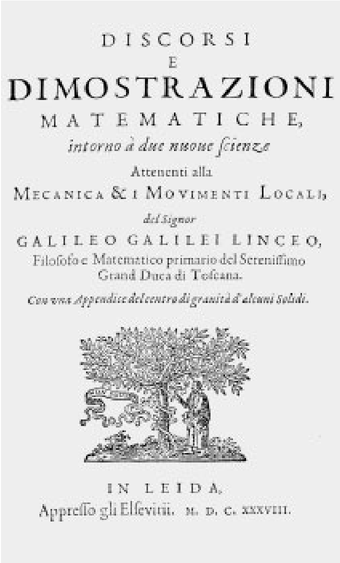


FIGURE 5-5
Cover of Galileo's important work *Dialogue Concerning Two New Sciences* (1638)

5.3.1 First day

Through the dialogue between Salviati and Sagredo, Galileo explains to the reader how engineering can play a great role as an object of natural science knowledge, i. e. that the analysis of the transformation of the engineering purpose-means relationship manifests itself in the form of the knowledge of the cause-effect relationship of the technically formulated nature. Salviati begins the dialogue: "The constant activity which you Venetians display in your famous arsenal suggests to the studious mind a large field for investigation, especially that part of the work which involves mechanics; for in this department all types of instruments and machines are constantly being constructed by many artisans, among whom there must be some who, partly by inherited experience and partly by their own observations, have become highly expert and clever in explanation." Sagredo replies: "You are quite right. Indeed, I myself, being curious by nature, frequently visit this place for the mere pleasure of observing the work of those who, on account of their superiority over other artisans, we call 'first rank men'. Conference with them has often helped me in the investigation of certain effects including not only those which are striking, but also those which are recondite and almost incredible. At times also I have been put to confusion and driven to despair of ever explaining something for which I could not account, but which my senses told me to be true …" [Galileo, 1638/1964, p. 3].

Such a causal relationship, which Galileo refers to again and again in examples on the first and second days of his *Dialogue*, represents the difference between the geometric and static similarity of objects in nature and engineering. "Therefore, Sagredo," says Salviati, "you would do well to change the opinion which you, and perhaps also many other students of mechanics, have entertained concerning the ability of machines and structures to resist external disturbances, thinking that when they are built of the same material and maintain the same ratio between parts, they are able equally, or rather proportionally, to resist or yield to such external disturbances and blows. For we can demonstrate by geometry that the large machine is not proportionately stronger than the small. Finally, we may say that, for every machine and structure, whether artificial or natural, there is set a necessary limit beyond which neither art nor nature can pass; it is here understood, of course, that the material is the same and the proportion preserved" [Galileo, 1638/1964, p. 5].

As will be shown below, Galileo's strength of materials investigations combine the question of the ultimate strength of simple loadbearing structures with the construction of transfer principles for such loadbearing structures. The latter is even today the task of the similarity mechanics founded by Galileo, which consists of deriving the mechanical behaviour of the large-scale construction by means of the mechanical findings acquired through experimentation on the model and the transfer principles. As Klaus-Peter Meinicke was able to show within the scope of his case study on the historical development of similarity theory, "Galileo's statements on similarity are a qualitative intervention in the scientific explanation of problems of scale transfer" [Meinicke, 1988, p. 15].

Salviati tries by way of qualitative examples to convince the others, Sagredo and Simplicio, that the geometric similarity may not be identified with the static. A cantilevering wooden stick that only just supports itself must break if it is enlarged; if the dimensions of the wooden stick are reduced, it would have reserves of strength. Galileo identifies here a very singular aspect of the collapse mechanism of loadbearing structures. But Sagredo and Simplicio still do not seem to have grasped the point; Salviati has to make things much clearer for them, and asks: "Who does not know that a horse falling from a height of three or four cubits will break his bones, while a dog falling from the same height or a cat from a height of eight or ten cubits will suffer no injury? Equally harmless would be the fall of a grasshopper from a tower or the fall of an ant from the distance of the moon" [Galileo, 1638/1964, p. 5]. Galileo quickly abandons such plausibility considerations appealing to "commonsense" which certainly contradict the premises of Galileo's similarity mechanics-based strength of materials considerations for the same material and geometric proportions. He describes how a marble obelisk supported at its ends on two timber baulks (statically determinate beam on two supports with span $l_0 = 2l$) fails exactly over the timber baulk added later in the middle (statically indeterminate beam on three supports with spans $l_1 = l_2 = l$) because one of the end supports has rotted away, i. e. the static system has been changed from a beam on three supports with one degree of static indeterminacy with spans $l_1 = l_2 = l$ into a statically determinate system with one span $l_1 = l$ and a cantilevering length $l_2 = l$. According to Galileo, yielding of one support in the original loadbearing system (statically determinate beam on two supports with span $l_0 = 2l$) would not have had any consequences, whereas for the beam on three supports with one degree of static indeterminacy with spans $l_1 = l_2 = l$ the force condition with an increasingly yielding support would be redistributed until the support reaction of the end support affected becomes zero, i. e. the statically indeterminate system has changed back to a statically determinate system, and the ensuing force condition causes the obelisk to fail over the (originally) central support. Galileo has thus identified the nature of the yielding support loading case for the simplest statically determinate system, but has not answered the question about the relationship between static and geometric similarity. It is therefore not surprising when even Sagredo, the intelligent layman, is not quite satisfied with Galileo's résumé that this would not have happened with a smaller but geometrically similar marble obelisk. Sagredo expresses his confusion: "… and I am the more puzzled because, on the contrary, I have noticed in other cases that the strength and resistance against breaking increase in a larger ratio than the amount of material. Thus, for instance, if two nails be driven into a wall, the one which is twice as big as the other will support not only twice as much weight as the other, but three or four times as much." To which Salviati replies: "Indeed you will not be far wrong if you say eight times as much …" [Galileo, 1638/1964, p. 7]. Thus, Galileo hints for the first time at the failure mechanism of the fixed-end beam. Sagredo: "Will you not then, Salviati, remove these difficulties and clear away these obscurities if possible? …" [Galileo, 1638/1964, p. 7].

Galileo begins his explanation with the tensile test (Fig. 5-6), "for this is the fundamental fact, involving the first and simple principle which we must take for granted as well known" [Galileo, 1638/1964, p. 7]. Galileo proposes a weight *C* of sufficient size that it breaks the cylinder of timber or other material where it is fixed at point *A*. Even non-fibrous materials such as stone or metal exhibit an ultimate strength. A distribution of the tensile resistance opposing the weight *C* over the area of the cross-section does not take place here. After Galileo attempts to explain qualitatively the tensile strength of a hemp rope whose fibres do not match the length of the test specimen by discussing the friction in the rope, he digresses from the explanation of the tensile resistance of non-fibrous materials and spends many pages discussing the "aversion of nature for empty space" [Galileo, 1638/1964, p. 11] and the cohesive force of the particles of such a body – these two together supposedly constitute said body's tensile resistance.

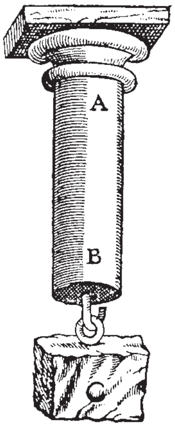


FIGURE 5-6
Galileo's tensile test as a thought experiment

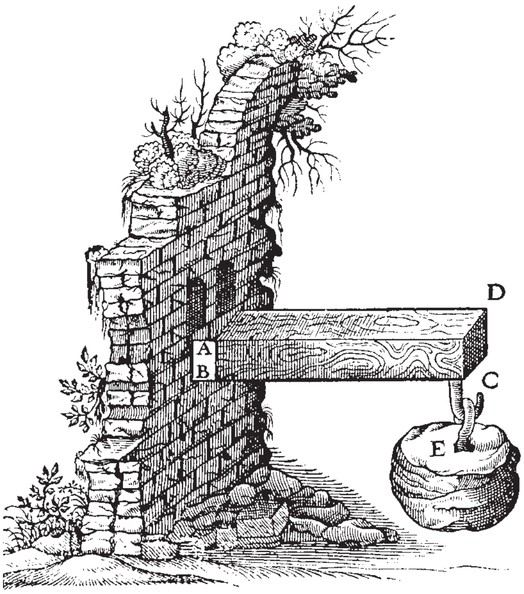
In this context, he answers the question regarding the breaking length of a copper wire quantitatively: "Take for instance a copper wire of any length and thickness; fix the upper end and to the other end attach a greater and greater load until finally the wire breaks; let the maximum load be, say, 50 pounds. Then it is clear that if 50 pounds of copper, in addition to the weight of the wire itself which may be, say, 1/8 ounce, is drawn out into wire of this same size we shall have the greatest length of this kind of wire which can sustain its own weight [breaking length – the author]. Suppose the wire which breaks to be one cubit in length and 1/8 ounce in weight; then since it supports 50 pounds in addition to its own weight, i. e. 4800 eighths of an ounce, it follows that all copper wires, independent of size, can sustain themselves up to a length of 4801 cubits and no more …" [Galileo, 1638/1964, p. 17]. This is one of the two places in the *Dialogue* where we could say that Galileo assumes a constant distribution of stress over the cross-section through the proportionality between tensile resistance and cross-sectional area.

Without having satisfied Sagredo's wish to discuss the failure problem of the fixed-end beam on the first day, Galileo instead devotes the major part of his discussion to a comprehensive explanation of mathematical questions and problems. On the first day of the *Dialogue* the reader gains the impression that Galileo's intention is to describe each question which will then be answered in detail the next day.

5.3.2 Second day

Salviati: "Resuming the thread of our discourse, whatever the nature of this resistance which solids offer to large tractive forces, there can at least be no doubt of its existence; and though this resistance is very great in the case of a direct pull, it is found, as a rule, to be less in the case of bending forces … It is this second type of resistance which we must consider,

FIGURE 5-7
Galileo's bending failure problem



seeking to discover in what proportion it is found in prisms and cylinders of the same material, whether alike or unlike in shape, length, and thickness" [Galileo, 1638/1964, p. 94]. The failure problem of the cantilever beam (beam fixed at one end) (Fig. 5-7) forms the true crux of Galileo's statements on the second day of his *Dialogue*.

After he has presented the lever principle and has distinguished clearly the force condition due to self-weight from one of "an immaterial body devoid of weight" [Galileo, 1638/1964, p. 96], he explains the collapse mechanism of the cantilever beam in three steps (Figs 5-8a to 5-8c):

a) The beam fails at B, which makes B the point of support and rotation of the kinematic collapse mechanism. Whereas the weight E_B acting at C exhibits the lever arm $\overline{BC} = l$, the tensile resistance W_Z at the fixed end acts at the lever arm $\overline{AB}/2 = h/2$, so that the beam can be idealised mechanically by the angle lever ABC (Fig. 5-8a).
b) From the tensile test (see Fig. 5-6), Galileo finds that the force at failure T_B is identical with the tensile resistance at the point of fixity W_Z (Fig. 5-8b).
c) Since $W_Z = T_B$, by applying the lever principle to the angle lever ABC, it follows that (Fig. 5-8c)

$$\frac{T_B}{E_B} = \frac{l}{\left(\frac{h}{2}\right)} \qquad (5\text{-}3)$$

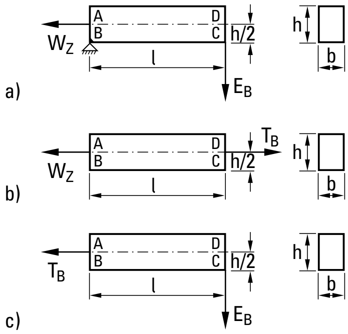


FIGURE 5-8
Schematic representation of Galileo's bending failure problem

As Galileo expressly notes, the steps a) to c) also apply when considering the self-weight of the prismatic cantilever beam. Like Leonardo, Galileo does not consider translation equilibrium at all; he should have applied the support reactions equivalent to T_B and E_B at B.

It has often been asked why Galileo applied the failure force $W_Z = T_B$ at the centre of gravity of the symmetrical beam cross-section and in doing so ignored the equilibrium conditions in the horizontal and vertical directions. This question can only be answered satisfactorily if we fully realise how crucial the tensile test was to his thought experiment. As the scientists and engineers prior to Newton (1643–1727) could only understand applied forces essentially in the form of weights, it is hardly surprising that Leonardo and Galileo thought of their test specimens being suspended from a fixed point. Acted upon by the force of gravity, the longitudinal axis of the specimen plus the suspended test weight aligned itself with the centre of the Earth. However, as in Galileo's tensile test (Fig. 5-6) the test weight is obviously introduced concentrically into the cylindrical cross-section via a hook at B and must always remain aligned with the centre of the Earth, the line of action coincides with the axis of the cylinder and hence also that of the tensile resistance at A. If we now analyse Galileo's cantilever beam, the axis of which is perpendicular to the direction of the force of gravity but is loaded at the end of the cantilever with a weight E_B, then the tensile resistance $W_Z = T_B$ must act in the axis of the centre of gravity of the beam cross-section because T_B was determined previously from a tensile test (Fig. 5-8b). It is important to realise that Galileo can imagine the force T_B in his tensile test only in the form of a weight that causes the concentric ultimate tensile force W_Z at the fixing point of the cross-section (compare Fig. 5-8b with Fig. 5-6). He imagines his test specimen loaded in tension and with its axis aligned with the direction of the centre of gravity now placed in a horizontal position, and instead of T_B applies the weight E_B perpendicular to the beam's centre-of-gravity axis, which results in the tensile force $W_Z = T_B$ at half the depth of the fixed-end cross-section $\overline{AB}$ (Figs. 5-8a and 5-8c). Galileo can determine the relationship between the ultimate tensile force $W_Z = T_B$ and the weight E_B using the lever principle applied to the angle lever ABC.

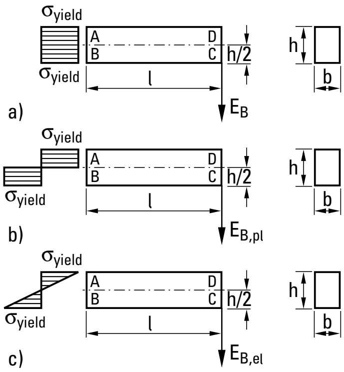


FIGURE 5-9
Comparison of the failure or ultimate loads: a) based on Galileo's stress theorem, b) with a fully plastic cross-section AB according to plastic hinge theory (see Fig. 5-1e), and c) when yielding starts at points A and B according to elastic theory.

Knowledge of the support reactions at B (Fig. 5-8a) is wholly unnecessary for solving this task. Whereas Leonardo's tensile test merely served to determine the tensile strength of various materials by way of experimentation, Galileo discovered in his beam problem the relationship between the tensile test and the static effect of the angle lever in the form of a proportion (see eq. 5-3). If we assume a constant stress distribution in Galileo's tensile test (Galileo never did this explicitly; from our modern viewpoint we can always assume a constant stress distribution when all the fibres parallel to the axis of the bar undergo the same change in length), i.e.

$$T_B = W_Z = \sigma_{yield} \cdot (b \cdot h) \tag{5-4}$$

(where σ_{yield} = yield stress of material, see Fig. 5-1e), then the failure load for the beam fixed at $\overline{AB}$ with length l, width b and depth h (Fig. 5-9a) is

$$E_B = \frac{\sigma_{yield}}{l} \cdot \frac{b \cdot h^2}{2} \tag{5-5}$$

As Galileo calculates the bending failure with eq. 5-3, from the modern viewpoint the comparison with the ultimate load

$$E_{B,pl} = \frac{\sigma_{yield}}{l} \cdot \frac{b \cdot h^2}{4} \tag{5-6}$$

for materials with distinct yield points and a fully plastic cross-section $\overline{AB}$ according to plastic hinge theory (Fig. 5-9b) would seem to apply, and not, as many authors still assume today, with

$$E_{B,el} = \frac{\sigma_{yield}}{l} \cdot \frac{b \cdot h^2}{6} \tag{5-7}$$

as the elastic ultimate load calculated from elastic theory at the start of yielding of the extreme top and bottom fibres at cross-section $\overline{AB}$ (Fig. 5-9c). (Many authors who have analysed Galileo's collapse mechanism for the cantilever beam compare eq. 5-7, which applies in the elastic range only – where the true stress $\sigma = \sigma_{yield}$ should be assumed instead of σ_{yield} – with eq. 5-5; but they are therefore comparing the serviceability limit state with the ultimate limit state.)

The ratio $E_B : E_{B,pl}$ is 2 : 1. This means that Galileo assumes the ultimate load to be twice the value of the maximum load according to plastic hinge theory. However, Galileo is interested in the ratios between the loads, and therefore statements about ultimate load ratios regarding "prisms and cylinders of the same material, whether alike or unlike in shape, length, and thickness" [Galileo, 1638/1964, p. 94]. What is still to be shown is that Galileo's ultimate load ratios are correct.

Firstly, Galileo calculates the ultimate ratio of a beam placed on edge (beam depth h, beam width b) to that of a beam placed flat (beam depth b, beam width h) of length l (Fig. 5-10). According to eq. 5-3, the following is valid for the beam placed on edge

$$\frac{T_B}{T} = \frac{l}{\left(\frac{h}{2}\right)} \tag{5-8}$$

and the following for the beam placed flat

$$\frac{T_B}{X} = \frac{l}{\left(\frac{b}{2}\right)} \tag{5-9}$$

Dividing eq. 5-9 by eq. 5-8 produces the following ultimate load ratio

$$\frac{T}{X} = \frac{h}{b} \tag{5-10}$$

After Galileo has proved that for the self-weight loading case the fixed-end moment of a prismatic cantilever beam is proportional to the square of its length, he goes on to analyse cantilever beams with a solid circular cross-section (Figs. 5-11a to 5-11c). Galileo begins by considering the tensile test for bars of diameters d_1 and D_1 with a solid circular cross-section (Fig. 5-11a). In doing so, he assumes that the ultimate tensile forces $T_{B,d}^2$ and $T_{B,D}^2$ are proportional to d_1^2 and D_1^2 respectively because "the [tensile] strength of the cylinder [with diameter D_1] is greater than that [with diameter d_1] in the same proportion in which the area of the circle [= cross-sectional area – the author] [with diameter D_1] exceeds that of circle [with diameter d_1]; because it is precisely in this ratio that the number of fibres binding the parts of the solid together in the one cylinder exceeds that in the other cylinder" [Galileo, 1638/1964, p. 100]. After equating the ultimate tensile forces $T_{B,d}^2$ and $T_{B,D}^2$ with the tensile resistances at the point of fixity of the two beams (Fig. 5-11b), eq. 5-3 takes on the following form:

$$\frac{T_{B,d}^2}{E_{B,d}^2} = \frac{l_1}{\left(\frac{d_1}{2}\right)} \tag{5-11}$$

$$\frac{T_{B,D}^2}{E_{B,D}^2} = \frac{l_1}{\left(\frac{D_1}{2}\right)} \tag{5-12}$$

Dividing eq. 5-12 by eq. 5-11 while taking into account the ultimate tensile forces $T_{B,d}^2$ and $T_{B,D}^2$ with the square of the diameter d_1 or D_1 produces an ultimate load ratio of

$$\frac{E_{B,d}^2}{E_{B,D}^2} = \frac{d_1^3}{D_1^3} \tag{5-13}$$

In the third step, Galileo varies the length of the cantilever beam as well as the diameter (Fig. 5-11c). From the proportionality of the ultimate tensile forces to the squares of the diameters

$$T_{B,d}^3 \sim d_2^2 \quad \text{or} \quad T_{B,D}^3 \sim D_2^2 \tag{5-14}$$

plus the moment equilibrium with respect to point B (see Fig. 5-8)

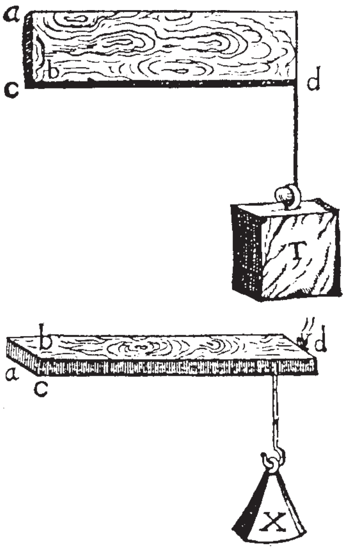


FIGURE 5-10
Galileo's consideration of the ultimate load ratio of a beam on edge (top) to one placed flat (bottom), (depth h or b, and width b or h depend on the respective orientation of the beam).

$$T^3_{B,d} \cdot \frac{d_2}{2} = E^3_{B,d} \cdot l_2 \quad \text{or}$$

$$T^3_{B,D} \cdot \frac{D_2}{2} = E^3_{B,D} \cdot L_2 \tag{5-15}$$

we get the following proportional relationships

$$E^3_{B,d} \cdot l_2 \sim d_2^3 \quad \text{or} \quad E^3_{B,D} \cdot L_2 \sim D_2^3 \tag{5-16}$$

and therefore the ultimate load ratio becomes

$$\frac{E^3_{B,d}}{E^3_{B,D}} = \frac{d_2^3}{D_2^3} \cdot \frac{L_2}{l_2} \tag{5-17}$$

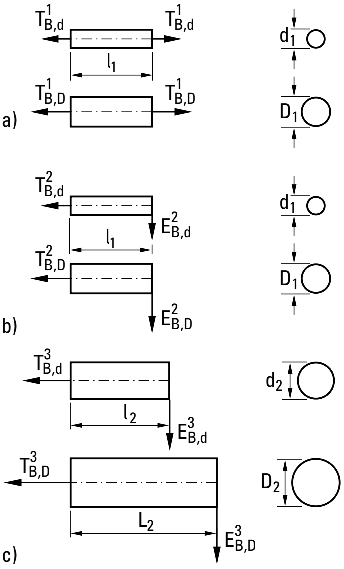


FIGURE 5-11
a) Tensile tests on bars of equal length but different diameters, plus the ultimate load ratios of b) cantilever beams of equal length but different diameters, and c) cantilever beams with unequal lengths and different diameters.

For geometrical similarity in particular, i.e. when

$$\frac{d_2}{D_2} = \frac{l_2}{L_2} = k = const. \tag{5-18}$$

is satisfied, eq. 5-17, taking into account the moment equilibrium of eq. 5-15, becomes

$$\frac{E^3_{B,d}}{E^3_{B,D}} \cdot \frac{l_2}{L_2} = \frac{M^3_{B,d}}{M^3_{B,D}} = k^3 = \left[\frac{T^3_{B,d}}{T^3_{B,D}} \right]^{\frac{3}{2}} \tag{5-19}$$

In response to eq. 5-19, Simplicio is astonished: "This proposition strikes me as both new and surprising: at first glance it is very different from anything which I myself should have guessed: for since these figures are similar in all other respects, I should have certainly thought that the forces and the resistances of these cylinders would have borne to each other the same ratio" [Galileo, 1638/1964, p. 104]. Salviati consoles him with the fact that he, too, at some stage also thought the resistances of similar cylinders would be similar, "but a certain casual observation showed me that similar solids do not exhibit a strength which is proportional to their size, the larger ones being less fitted to undergo rough usage …" [Galileo, 1638/1964, p. 104].

And Galileo went further. He proves the assertion of the first day that among geometrically similar prismatic cantilever beams there is only one "which under the stress of its own weight lies just on the limit between breaking and not breaking: so that every larger one is unable to carry the load of its own weight and breaks; while every smaller one is able to withstand some additional force tending to break it" [Galileo, 1638/1964, p. 105].

After Galileo has also solved the task of calculating – for a cantilever beam of given length with a solid circular cross-section – the diameter at which, subjected to its own weight only, the ultimate limit state is reached, he can sum up: "From what has already been demonstrated, you can plainly see the impossibility of increasing the size of structures to vast dimensions either in art or in nature; likewise the impossibility of building ships, palaces, or temples of enormous size in such a way that their oars, yards, beams, iron-bolts, and, in short, all their other parts will hold together; nor can nature produce trees of extraordinary size because the branches would break down under their own weight …" [Galileo, 1638/1964, p. 108]. The conclusion of Galileo's similarity mechanics-based strength considerations regarding the cantilever beam is – analogous to the breaking length of the bar in tension – the answer to the question of the maximum length of a cantilever beam loaded by its own weight only.

At the start of his examination of the symmetrical lever and the beam on two supports (Fig. 5-12), Galileo verifies that such beams may be twice as long as a cantilever beam. From that it follows that the fixed-end moment of the cantilever beam is equivalent to the maximum span moment of the beam on two supports and also the support moment of the symmetrical lever. Afterwards, he specifies the proportions of the ultimate limit states of the symmetrical with respect to the asymmetrical lever: "Another rather interesting problem may be solved as a consequence of this theorem, namely, given the maximum weight which a cylinder or prism can support at its middle-point where the resistance is a minimum, and given also a larger weight, find that point in the cylinder for which this larger weight is the maximum load that can be supported" [Galileo, 1638/1964, p. 115].

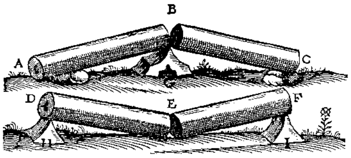


FIGURE 5-12
Failure mechanisms of the symmetrical lever and the beam on two supports (after Galileo)

The problem is illustrated in Figs 5-13a and b. (Without mentioning this explicitly, Galileo releases the beam on two supports and introduces a support at each point of load application E_B or E'_B; he has thus reduced the problem to the statically equivalent systems of the symmetrical or asymmetrical lever.) From the equivalence of the ultimate moments

$$M_{B,E} = 0.25 \cdot l \cdot E_B = M_{B,E'} = E'_B \cdot \frac{a \cdot b}{l} \qquad (5\text{-}20)$$

it follows that the ratio of the ultimate loads is

$$\frac{E'_B}{E_B} = \frac{l^2}{4 \cdot a \cdot b} \qquad (5\text{-}21)$$

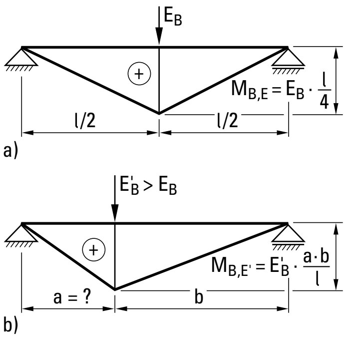


FIGURE 5-13
Bending moment diagram for a beam on two supports at the ultimate limit state for
a) central maximum point load, and
b) eccentric maximum point load

Galileo now turns his attention once again to the cantilever beam. He poses the question of which longitudinal form such a beam must have when loaded with a point load at the end of the cantilever so that the ultimate limit state is reached at every cross-section. Galileo proves that the longitudinal form must be that of a quadratic parabola (Fig. 5-14), and immediately identifies the engineering advantages: "It is thus seen how one can diminish the weight of a beam by as much as thirty-three per cent without diminishing its strength; a fact of no small utility in the construction of large vessels, and especially in supporting the decks, since in such structures lightness is of prime importance" [Galileo, 1638/1964, p. 118]. He provides the practical builder with methods for constructing parabolas – theoretically correct, but less than practicable, and theoretically incorrect, but a good approximation.

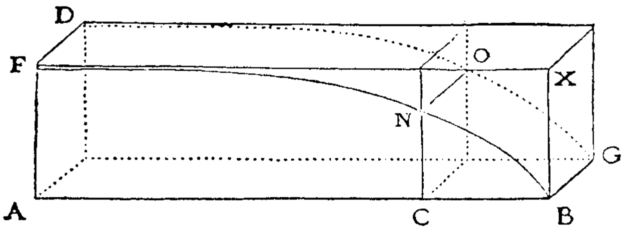


FIGURE 5-14
A beam fixed at AFD with a point load at the end of the cantilever BG must have the longitudinal form of a quadratic parabola (FNBGOD) if the ultimate limit state is to be reached at every cross-section (after Galileo).

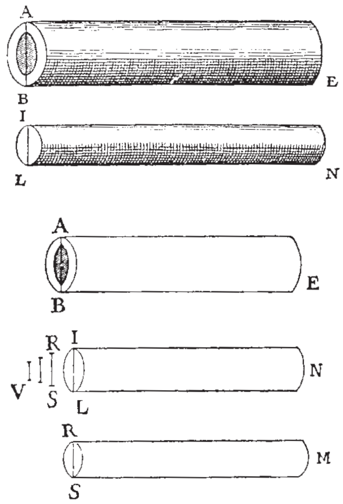


FIGURE 5-15
Relationship between the ultimate loads of cantilever beams with solid and hollow circular cross-sections fixed at the left-hand end (after Galileo)

At the end of the second day of the *Dialogue*, Galileo also investigates cantilever beams with annular cross-sections "for the purpose of greatly increasing strength without adding to weight" [Galileo, 1638/1964, p. 123] compared to cantilever beams with a solid circular cross-section.

He discovers that the relationship between the ultimate loads is the same as that between the diameters $\overline{AB}$ and $\overline{IL}$ (Fig. 5-15). Using this proportion and the eq. 5-13 valid for the cantilever beam with solid circular cross-section (diameters $\overline{IL}$ and $\overline{RS}$), Galileo deduces that the ratio between the ultimate loads at *E* and *M* must be the same as for the product of $\overline{IL}^2 \cdot \overline{AB}$ to $\overline{RS}^3$.

This deduction ends the second day of Galileo's *Dialogue* and hence also his similarity mechanics-based theory of the failure of simple beam-type loadbearing systems. It almost sounds like Galileo is commanding future generations of scientists to carry out further research when he says (through Salviati): "Hitherto we have demonstrated numerous conclusions pertaining to the resistance which solids offer to fracture. As a starting point for this science, we assumed that the resistance offered by the solid to a straight-away pull was known; from this base one might proceed to the discovery of many other results and their demonstrations; of these results the number to be found in nature is infinite" [Galileo, 1638/1964, p. 123].

5.4 Developments in the strength of materials up to 1750

In almost all the history of science works dealing with the development of beam theory from Galileo to Navier (1785–1836), the bending failure problem of Galileo (Fig. 5-7) and its proposed solution is interpreted as though Galileo was already aware of the notion of stress. However, this notion, crucial to strength of materials, was not generally defined until 1823 – by the civil engineer and mathematician A. L. Cauchy (1789–1857) after he had employed the limit state notion of d'Alembert for the theoretical foundation of differential and integral calculus two years before. Stress, too, is a limiting value because the divided difference $\Delta P/\Delta F$ (ΔP is the internal force that acts on a finite area ΔF) translates to the differential quotient dP/dF for ever smaller areas ΔF, i. e. ΔF tries to attain zero. The prerequisite for the establishment of this notion was that the solid body under consideration should not consist of a finite number of indivisible elements of finite size such as atoms or molecules, but rather that the material be distributed throughout the solid body (continuum hypothesis). The continuum hypothesis, too, did not advance to become the generally accepted structural model of the body in the emerging fundamental engineering science discipline of strength of materials until long after Euler's work on hydromechanics (1749) and Cauchy's founding of continuum mechanics in the 1820s [Truesdell, 1968, pp. 123–124]. As could be shown, Galileo's embryonic strength of materials theory was limited to the knowledge that the ultimate tensile forces from tensile tests were related to the geometrically similar cantilever beams abstracted to angle levers at failure. It was only for this reason that Galileo could ignore the relationship between force and deformation conditions conveyed by the material law in his tensile test and beam problem.

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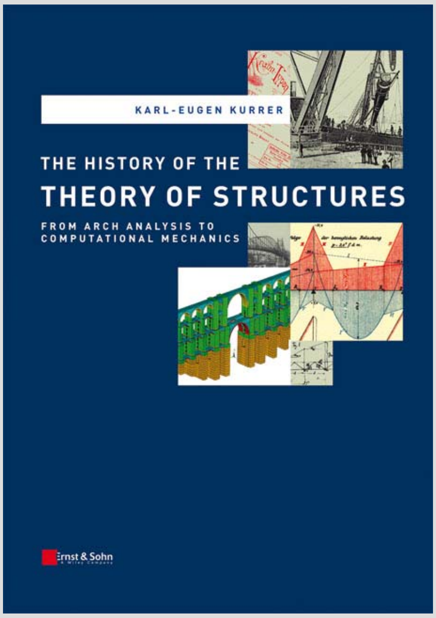

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