

Karl-Eugen Kurrer

The History of the Theory of Structures

Searching for Equilibrium

- unverzichtbare Hilfe in der Planungspraxis
- normenähnlicher Charakter
- die Empfehlungen werden auch in Ausschreibungen und Abrechnungen verwendet

Wann setzte das statische Rechnen im Entwurfsprozess ein? Beginnend mit den Festigkeitsbetrachtungen von Leonardo und Galilei wird die Herausbildung baustatischer Verfahren vorgestellt. Neu in der 2. Aufl.: Erddrucktheorie, Schalentheorie, Computerstatik, FEM, 260 Kurzbiografien.

BESTELLEN

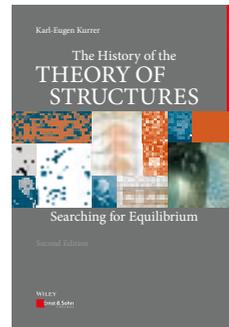
+49 (0)30 470 31-236

marketing@ernst-und-sohn.de

www.ernst-und-sohn.de/3229

WILEY

Ernst & Sohn
A Wiley Brand



2018 · 1212 Seiten · 1002 Abbildungen

Hardcover

ISBN 978-3-433-03229-9

€ 149*

WILEY Ernst & Sohn
A Wiley Brand

ÜBER DAS BUCH

Zehn Jahre nach der 1. Auflage in englischer Sprache legt der Autor sein Buch *The History of the Theory of Structures* in wesentlich erweiterter Form vor, nunmehr mit dem Untertitel *Searching for Equilibrium*. Mit dem vorliegenden Buch lädt der Verfasser seine Leser zur Suche nach dem Gleichgewicht von Tragwerken auf Zeitreisen ein. Die Zeitreisen setzen mit der Entstehung der Statik und Festigkeitslehre eines Leonardo und Galilei ein und erreichen ihren ersten Höhepunkt mit den baustatischen Theorien über den Balken, Erddruck und das Gewölbe von Coulomb am Ende des 18. Jahrhunderts. Im folgenden Jahrhundert formiert sich die Baustatik mit Navier, Culmann, Maxwell, Rankine, Mohr, Castigliano und Müller-Breslau zu einer technikwissenschaftlichen Grundlagendisziplin, die im 20. Jahrhundert in Gestalt der modernen Strukturmechanik bei der Herausbildung der konstruktiven Sprache des Stahl-, Stahlbeton-, Flugzeug-, Automobil- und des Schiffbaus eine tragende Rolle spielt. Dabei setzt der Autor den inhaltlichen Schwerpunkt auf die Formierung und Entwicklung moderner numerischer Ingenieurmethoden wie der Finite-Elemente-Methode und beschreibt ihre

disziplinäre Integration in der Computational Mechanics. Kurze, durch historische Skizzen unterstützte Einblicke in gängige Berechnungsverfahren erleichtern den Zugang zur Geschichte der Strukturmechanik und Erd-drucktheorie vom heutigen Stand der Ingenieurpraxis und stellen einen auch einen wichtigen Beitrag zur Ingenieurpädagogik dar.

Dem Autor gelingt es, die Unterschiedlichkeit der Akteure hinsichtlich ihres technisch-wissenschaftlichen Profils und ihrer Persönlichkeit plastisch zu schildern und das Verständnis für den gesellschaftlichen Kontext zu erzeugen. So werden in 260 Kurzbiografien die subjektive Dimension der Baustatik und der Strukturmechanik von der frühen Neuzeit bis heute entfaltet. Dabei werden die wesentlichen Beiträge der Protagonisten der Baustatik besprochen und in die nachfolgende Bibliografie integriert. Berücksichtigt wurden nicht nur Bauingenieure und Architekten, sondern auch Mathematiker, Physiker, Maschinenbauer sowie Flugzeug- und Schiffbauer. Das vorliegende Buch ist die erste zusammenfassende historische Gesamtdarstellung der Baustatik vom 16. Jahrhundert bis heute.

BESTELLUNG

| Anzahl | ISBN / | Titel | Preis |
|--------|-------------------|---|--------|
| | 978-3-433-03229-9 | The History of the Theory of Structures | € 149* |

Privat

Geschäftlich

Bitte richten Sie Ihre Bestellung an:

Tel. +49 (0)30 47031-236

Fax +49 (0)30 47031-240

marketing@ernst-und-sohn.de

Firma

UST-ID Nr.

Name, Vorname

Telefon

Fax

Straße, Nr.

PLZ/Ort/Land

E-Mail

www.ernst-und-sohn.de/3229

Datum/Unterschrift

Foreword of the series editors

Construction history has experienced amazing momentum over the past decades. It has become a highly vibrant, independent discipline attracting much attention through its international networks. Although research projects at national level focus on different themes, they are united through the knowledge that their diversity in terms of content and methods, and hence the associated synthesizing potential, are precisely the strengths that shape this new field of research. Construction history opens up new ways of understanding construction between engineering and architecture, between the history of building and history of art, between the history of technology and history of science. Since the appearance of the first German edition in 2002, *The History of the Theory of Structures* has become a standard work of reference for this latter field. It continues the series of great works on the history of civil and structural engineering by S. P. Timoshenko and I. Szabó right up to E. Benvenuto and J. Heyman, and enriches them by adding valuable new levels of interpretation and knowledge. We are delighted to be able to publish the second, considerably enlarged, English-language edition as part of the *Construction History Series/ Edition Bautechnikgeschichte*.

Werner Lorenz and Karl-Eugen Kurrer
Series editors

Foreword

Ten years after the first English edition of Dr. Kurrer's *The History of the Theory of Structures*, he now presents us with a much enlarged edition, and with a new subtitle: *Searching for Equilibrium* – an addition that reminds us of that most important of all mechanical principles: no equilibrium, no loadbearing system! But the subtitle also expresses the constant search for a balance between theory of structures as a scientific discipline and its prime task in practical applications – totally in keeping with Leibniz' *Theoria cum Praxi*. This interaction has proved beneficial for both sides at all times in history, and runs like a thread through the entire book.

New content in this second edition includes: earth pressure theory, ultimate load method, an analysis of historical textbooks, steel bridges, light-weight construction, plate and shell theory, computational statics, Green's functions, computer-assisted graphical analysis and historical engineering science. Furthermore, the number of brief biographies has been increased from 175 to 260! Compared with the first English edition, the number of printed pages has increased by 50 % to a little over 1,200.

Right at the start we learn that the first conference on the history of theory of structures took place in Madrid in 2005. This theme, its parts dealt with many times, is simply crying out for a comprehensive treatment. However, this book is not a history book in which the contributions of our predecessors to this theme are listed chronologically and described systematically. No, this is 'Kurrer's History of Theory of Structures' with his interpretations and classifications; luckily – because that makes it an exciting journey through time, with highly subjective impressions, more thematic and only roughly chronological, and with a liking for scientific theory. Indeed, a description of the evolution of an important fundamental engineering science discipline with its many facets in teaching, research and, first and foremost, practice.

And what is "theory of structures" anyway? ... Gerstner's first book dating from 1789 talks about the "statics of architecture" and Emil Winkler used the term "statics of structures" around 1880. Winkler's term also included earth pressure theory, the evolution of which from 1700 to the present day is now the topic of a new chapter 5 in this second edition.

The history of theory of structures is in the first place the history of mechanics and mathematics, which in earlier centuries were most definitely understood to be applied sciences. Dr. Kurrer calls this period from 1575 to 1825 the "preparatory period" – times in which structural design was still very much dominated by empirical methods. Nevertheless, it is worth noting that the foundations of many structural theories were laid

in this period. It is generally accepted that the structural report for the repairs to the dome of St. Peter's in Rome (1742/1743) by the *tre matematici* represents the first structural calculations as we understand them today. In other words, dealing with a constructional task by the application of scientific methods – accompanied, characteristically, by the eternal dispute between theory and practice (see section 13.2.5). These days, the centuries-old process of the theoretical abstraction of natural and technical processes in almost all scientific disciplines is called ‘modelling and simulation’ – as though it had first been introduced with the invention of the computer and the world of IT, whereas, in truth, it has long since been the driving force behind humankind's ideas and actions. Mapping the load-bearing properties of building structures in a theoretical model is a typical case. Classic examples are the development of masonry and elastic arch theories (see chapter 4) and the continuum mechanics models of earth pressure of Rankine and Boussinesq (see sections 5.4 and 5.5). It has become customary to add the term ‘computational’ to these computer-oriented fields in the individual sciences, in this case ‘computational mechanics’.

The year 1825 has been fittingly chosen as the starting point of the discipline-formation period in theory of structures (see chapter 7). Theory of structures is not just the solving of an equilibrium problem, not just a computational process. Navier, whose importance as a mechanics theorist we still acknowledge today in the names of numerous theories (Navier stress distribution, Navier-Lamé and Navier-Stokes equations, etc.), was very definitely a practitioner. In his position as professor for applied mechanics at the *École des Ponts et Chaussées*, it was he who combined the subjects of applied mechanics and strength of materials in order to apply them to the practical tasks of building. For example, in his *Mechanik der Baukunst* of 1826, he describes the work of engineers thus: “... after the works have been designed and drawn, [they] investigate them to see if all conditions have been satisfied and improve their design until this is the case. Economy is one of the most important conditions here; stability and durability are no less important ...” (see section 2.1.2.1). Navier was the first to establish theory of structures as an independent scientific discipline. Important structural theories and methods of calculation would be devised in the following years, linked with names such as Clapeyron, Lamé, Saint-Venant, Rankine, Maxwell, Cremona, Castigliano, Mohr and Winkler, to name but a few. The graphical statics of Culmann and its gradual development into graphical analysis are milestones in the history of theory of structures.

Already at this juncture, it is worth pointing out that the development did not always proceed smoothly – controversies concerning the content of theories, or competition between disciplines, or priority disputes raised their heads along the way. This exciting theme is explored in detail in chapter 13 by way of 13 examples.

In the following decades, the evolution of methods in theory of structures became strongly associated with specific structural systems and hence, quite naturally, with the building materials employed, such as iron

(steel) and later reinforced concrete (see chapters 8, 9 and 10). Independent materials-specific systems and methods were devised. Expressed in simple terms, structural steelwork, owing to its modularity and the fabrication methods, initially concentrated on assemblies of linear members, not embracing plate and shell structures until the 1950s. On the other hand, reinforced concrete preferred its own two-dimensional design language, which manifested itself in slabs, plates and shells. Therefore, chapters 8 and 10 in this second English edition have been considerably enlarged by the addition of plate and shell structures. The space frames dealt with in chapter 9 represent a link to some extent. This materials-based split was also reflected in the teaching of theory of structures in the form of separate studies. It was not until many years later that the parts were brought together in a homogeneous theory of structures, albeit frequently 'neutralised', i. e. no longer related to the specific properties of the particular building material – an approach that must be criticised in retrospect. Of course, the methods of structural analysis can encompass any material in principle, but in a specific case they must take account of the particular characteristics of the material.

Dr. Kurrer places the transition from the discipline-formation period – with its great successes in the shape of graphical statics and the systematic approach to methods of calculation in member analysis in the form of the force method – to the consolidation period around 1900. This latter period, which lasted until 1950, is characterised by refinements and extensions, e. g. a growing interest in plate and shell structures and the consideration of non-linear effects. Only after this does the 'modern' age of theory of structures begin – designated the integration period in this instance and typified by the use of modern computers and powerful numerical methods. Theory of structures is integrated into the structural planning process of draft design – analysis – detailed design – construction in this period. Have we reached the end of the evolutionary road? Does this development mean that theory of structures, as an independent engineering science, is losing its profile and its justification? The tendencies of recent years indicate the opposite.

The story of yesterday and today is also the story of tomorrow. In the world of data processing and information technology, theory of structures has undergone rapid progress in conjunction with numerous paradigm changes. It is no longer the calculation process and method issues, but rather principles, modelling, realism, quality assurance and many other aspects that form the focus of our attention. The remit includes dynamics alongside statics; in terms of the role they play, plate and shell structures are almost equal to trusses, and taking account of true material behaviour is obligatory these days. During its history so far, theory of structures was always the trademark of structural engineering; it was never the discipline of 'number crunchers', even if this was and still is occasionally proclaimed as such when launching relevant computer programs. Theory of structures continues to play an important mediating role between mechanics on the one side and the draft and detailed design subjects on the other side

in teaching, research and practice. Statics and dynamics have in the meantime advanced to what is known internationally as ‘computational structural mechanics’, a modern application-related structural mechanics.

The author takes stock of this important development in chapters 11 and 12. He mentions the considerable rationalisation and formalisation – the foundations for the subsequent automation. It was no surprise when, as early as the 1930s, the structural engineer Konrad Zuse began to develop the first computer (see section 11.4). However, the rapid development of numerical methods for structural calculations in later years could not be envisaged at that time. J. H. Argyris, one of the founding fathers of the modern finite element method, recognised this at an early stage in his visionary remark “the computer shapes the theory” (1965): Besides theory and experimentation, there is a new pillar – numerical simulation (see section 12.1).

By their very nature, computers and programs have revolutionised the work of the structural engineer. Have we not finally reached the stage where we are liberated from the craftsman-like, formula-based business so that we can concentrate on the essentials? The role of modern theory of structures is discussed in section 14.1, also in the context of the relationship between the structural engineer and the architect. A new graphical statics has appeared, not in the sense of the automation and visual presentation of Culmann’s graphical statics, but rather in the form of graphic displays and animated simulations of mechanical relationships and processes. This is a decisive step towards the evolution of structures and to load-bearing structure synthesis, to a new way of teaching structural engineering (see section 14.1.4). This potential as a living interpretation and design tool has not yet been fully exploited. It is also worth mentioning that the boundaries to the other construction engineering disciplines (mechanical engineering, automotive engineering, shipbuilding, aerospace, biomechanics) are becoming more and more blurred in the field of computational mechanics; the relevant conferences no longer make any distinctions. The concepts, methods and tools are universal. And we are witnessing similar developments in teaching, too. No wonder Dr. Kurrer also refers to leading figures from these disciplines. That fact becomes particularly clear in chapter 15, which contains 260 brief biographies of persons who have featured prominently in the theory of structures.

In terms of quality and quantity, this second English edition of *The History of the Theory of Structures* goes way beyond the first edition. This book could only have been written by an expert, an engineer who knows the discipline inside out. Engineering scientists getting to grips with their own history so intensely is a rare thing. But this is one such lucky instance. We should be very grateful to Dr.-Ing. Dr.-Ing. E.h. Karl-Eugen Kurrer, and also ‘his’ publisher, Ernst & Sohn (John Wiley & Sons), for his *magnum opus*.

Stuttgart, February 2018
Ekkehard Ramm, University of Stuttgart

**Preface to the second
English edition**

Encouraged by the positive feedback from the engineering world regarding the first German edition of my *Geschichte der Baustatik* (2002) and the first English edition *The History of the Theory of Structures* (2008), two years ago I set myself the task of revising my manuscripts, adding new material once again and bringing everything up to date. Increasing the number of pages by a little over 50% was unavoidable, because my goal now was to present a total picture of the evolution of the theory of structures.

But that goal did not just consist of including the research findings of the past few years. Instead, I would now be devoting more space to a detailed treatment of the development of modern numerical methods of structural analysis and structural mechanics as well as the connection between the formation of structural analysis theories and constructional-technical progress. It is for this reason that, for example, plate, shell and stability theories have been paid particular attention, as these theories played an important part in the development of the design languages of steel, reinforced concrete, aircraft, vehicles and ships. As a result, the chapters on steel (chapter 8) and reinforced concrete (chapter 10) have been greatly enlarged. Without doubt, the finite element method (FEM), spawned by structural mechanics and numerical mathematics, was the most important intellectual technology of the second half of the 20th century. Therefore, the historico-logical sources of computational statics plus their development and establishment are now presented in detail separately in chapter 12. Also new is the substantial chapter on the 300-year-old history of earth pressure theory (chapter 5). Earth pressure theory was the first genuine engineering science theory that shaped the scientific self-conception of modern civil engineering, a profession that was beginning to emerge in 18th-century France. It is the reference theory for this profession, and not beam theory, as is often assumed. Not until the 20th century did earth pressure theory gradually become divorced from theory of structures. As in earth pressure theory, it is the search for equilibrium that grabs our historico-logical attention in masonry arch theory. Chapter 4, “From masonry arch to elastic arch”, has therefore been expanded. The same is true for chapter 3, which covers the development of theory of structures and applied mechanics as the first fundamental engineering science disciplines. That chapter not only contains the first analysis of textbooks on these two sciences published in the 19th and 20th centuries, but also attempts to extract the scientific and epistemological characteristics of theory of structures and applied mechanics. That therefore also forms the starting point for chapter 14, “Perspectives for a historical theory of structures”, the integral constituent of my concept for a historical engineering science, which is explained in detail in this book. Current research into graphical statics is one example mentioned in this chapter, which I summarise under the term “computer-aided graphic statics” (CAGS). The number of brief biographies of the protagonists of theory of structures and structural mechanics has increased by 85 to 260, and the bibliography also contains many new additions.

Probably the greatest pleasure during the preparation of this book was experiencing the support that my many friends and colleagues afforded me. I would therefore like to thank: Katherine Alben (Niskayuna, N.Y.), William Baker (Chicago), Ivan Baláz (Bratislava), Jennifer Beal (Chichester), Norbert Becker (Stuttgart), Antonio Becchi (Berlin), Alexandra R. Brown (Hoboken), José Calavera (Madrid), Christopher R. Calladine (Cambridge, UK), Kostas Chatzis (Paris), Mike Chrimes (London), Ilhan Citak (Lehigh), Zbigniew Cywiński (Gdańsk), René de Borst (Delft), Giovanni Di Pasquale (Florence), Cengiz Dicleli (Constance), Werner Dirschmid (Ingolstadt), Albert Duda (Berlin), Holger Eggemann (Brühl), Bernard Espion (Brussels), Jorun Fahle (Gothenburg), Amy Flessert (Minneapolis), Hubert Flomenhoft (Palm Beach Gardens), Peter Groth (Pfullingen), Carl-Eric Hagentoft (Gothenburg), Friedel Hartmann (Kassel), Hans-Joachim Haubold (Darmstadt), Eva Haubold-Marguerre (Darmstadt), Torsten Hoffmeister (Berlin), Santiago Huerta (Madrid), Peter Jahn (Kassel), Andreas Kahlow (Potsdam), Christiane Kaiser (Potsdam), Sándor Kaliszky (Budapest), Andreas Kirchner (Würzburg), Klaus Knothe (Berlin), Winfried B. Krätzig (Bochum), Arnold Krawietz (Berlin), Eike Lehmann (Lübeck), Werner Lorenz (Cottbus/Berlin), Andreas Luetjen (Braunschweig), Stephan Luther (Chemnitz), René Maquoi (Liège), William J. Maher (Urbana), Gleb Mikhailov (Moscow), Juliane Mikoletzky (Vienna), Klaus Nippert (Karlsruhe), John Ochsendorf (Cambridge, Mass.), Eberhard Pelke (Mainz), Christian Petersen (Ottobrunn), Ines Prokop (Berlin), Frank Purtak (Dresden), Ekkehard Ramm (Stuttgart), Patricia Radelet-de Grave (Louvain-la-Neuve), Anette Rühlmann (London), Jan Peter Schäfermeyer (Berlin), Lutz Schöne (Rosenheim), Sabine Schroyen (Düsseldorf), Luigi Sorrentino (Rome), Valery T. Troshchenko (Kiev), Stephanie Van de Voorde (Brussels), Volker Wetzck (Cottbus), Jutta Wiese (Dresden), Erwin Wodarczak (Vancouver) and Ine Wouters (Brussels).

I am indebted to the technical and design skills of Sophie Bleifuß (typodesign), Siegmur Hiller (production), Uta-Beate Mutz (typesetting) and Peter Palm (drawings), who together helped to guarantee a high-quality production. And without the great support of my family, this book would have been impossible. My dear wife and editor, Claudia Ozimek, initiated the project at the Ernst & Sohn publishing house, and it was my colleague Ute-Marlen Günther who steered the project safely to a successful conclusion. Finally, I would like to thank all my colleagues at Ernst & Sohn who have supported this project and who are involved in the distribution of my book.

I hope that you, dear reader, will be able to absorb the knowledge laid out in this book and not only benefit from it, but also simply enjoy the learning experience.

Berlin, March 2018

Karl-Eugen Kurrer

Contents

| | |
|--------|--|
| V | Foreword of the series editors |
| VI | Foreword |
| X | Preface to the second English edition |
| XXVI | About this series |
| XXVII | About the series editors |
| XXVIII | About the author |
| 2 | 1 The tasks and aims of a historical study of the theory of structures |
| 4 | 1.1 Internal scientific tasks |
| 8 | 1.2 Practical engineering tasks |
| 9 | 1.3 Didactic tasks |
| 11 | 1.4 Cultural tasks |
| 12 | 1.5 Aims |
| 12 | 1.6 An invitation to take part in a journey through time to search for the equilibrium of loadbearing structures |
| 14 | 2 Learning from history: 12 introductory essays |
| 15 | 2.1 What is theory of structures? |
| 15 | 2.1.1 Preparatory period (1575–1825) |
| 15 | 2.1.1.1 Orientation phase (1575–1700) |
| 17 | 2.1.1.2 Application phase (1700–1775) |
| 17 | 2.1.1.3 Initial phase (1775–1825) |
| 18 | 2.1.2 Discipline-formation period (1825–1900) |
| 19 | 2.1.2.1 Constitution phase (1825–1850) |
| 20 | 2.1.2.2 Establishment phase (1850–1875) |
| 21 | 2.1.2.3 Classical phase (1875–1900) |
| 22 | 2.1.3 Consolidation period (1900–1950) |
| 22 | 2.1.3.1 Accumulation phase (1900–1925) |
| 23 | 2.1.3.2 Invention phase (1925–1950) |
| 24 | 2.1.4 Integration period (1950 to date) |
| 25 | 2.1.4.1 Innovation phase (1950–1975) |
| 26 | 2.1.4.2 Diffusion phase (1975 to date) |
| 27 | 2.2 From the lever to the trussed framework |
| 27 | 2.2.1 Lever principle according to Archimedes |

| | | |
|----|---------|--|
| 28 | 2.2.2 | The principle of virtual displacements |
| 28 | 2.2.3 | The general work theorem |
| 29 | 2.2.4 | The principle of virtual forces |
| 29 | 2.2.5 | The parallelogram of forces |
| 30 | 2.2.6 | From Newton to Lagrange |
| 31 | 2.2.7 | The couple |
| 32 | 2.2.8 | Kinematic or geometric school of statics? |
| 33 | 2.2.9 | Stable or unstable, determinate or indeterminate? |
| 33 | 2.2.10 | Syntheses in statics |
| 36 | 2.2.11 | Schwedler's three-pin frame |
| 38 | 2.3 | The development of higher engineering education |
| 39 | 2.3.1 | The specialist and military schools of the <i>ancien régime</i> |
| 40 | 2.3.2 | Science and enlightenment |
| 40 | 2.3.3 | Science and education during the French Revolution (1789–1794) |
| 41 | 2.3.4 | Monge's curriculum for the École Polytechnique |
| 42 | 2.3.5 | Austria, Germany and Russia in the wake of the École Polytechnique |
| 46 | 2.3.6 | The education of engineers in the United States |
| 51 | 2.4 | A study of earth pressure on retaining walls |
| 53 | 2.4.1 | Earth pressure determination according to Culmann |
| 54 | 2.4.2 | Earth pressure determination according to Poncelet |
| 55 | 2.4.3 | Stress and stability analyses |
| 58 | 2.5 | Insights into bridge-building and theory of structures in the 19th century |
| 58 | 2.5.1 | Suspension bridges |
| 60 | 2.5.1.1 | Austria |
| 61 | 2.5.1.2 | Bohemia and Moravia |
| 62 | 2.5.1.3 | Germany |
| 63 | 2.5.1.4 | United States of America |
| 64 | 2.5.2 | Timber bridges |
| 67 | 2.5.3 | Hybrid systems |
| 68 | 2.5.4 | The Göltzsch and Elster viaducts (1845–1851) |
| 70 | 2.5.5 | The Britannia Bridge (1846–1850) |
| 73 | 2.5.6 | The first Dirschau Bridge over the Vistula (1850–1857) |
| 75 | 2.5.7 | The Garabit Viaduct (1880–1884) |
| 79 | 2.5.8 | Bridge engineering theories |
| 80 | 2.5.8.1 | Reichenbach's arch theory |
| 81 | 2.5.8.2 | Young's masonry arch theory |
| 84 | 2.5.8.3 | Navier's suspension bridge theory |
| 85 | 2.5.8.4 | Navier's <i>Résumé des Leçons</i> |
| 86 | 2.5.8.5 | The trussed framework theories of Culmann and Schwedler |
| 87 | 2.5.8.6 | Beam theory and stress analysis |
| 88 | 2.6 | The industrialisation of steel bridge-building between 1850 and 1900 |
| 88 | 2.6.1 | Germany and Great Britain |
| 90 | 2.6.2 | France |
| 92 | 2.6.3 | United States of America |
| 97 | 2.7 | Influence lines |
| 97 | 2.7.1 | Railway trains and bridge-building |
| 99 | 2.7.2 | Evolution of the influence line concept |

| | | |
|-----|----------|--|
| 101 | 2.8 | The beam on elastic supports |
| 102 | 2.8.1 | The Winkler bedding |
| 102 | 2.8.2 | The theory of the permanent way |
| 104 | 2.8.3 | From permanent way theory to the theory of the beam on elastic supports |
| 106 | 2.8.4 | Geotechnical engineering brings progress |
| 107 | 2.9 | Displacement method |
| 108 | 2.9.1 | Analysis of a triangular frame |
| 109 | 2.9.1.1 | Bar end moments |
| 110 | 2.9.1.2 | Restraint forces |
| 112 | 2.9.1.3 | Superposition means combining the state variables linearly with the solution |
| 112 | 2.9.2 | Comparing the displacement method and trussed framework theory for frame-type systems |
| 113 | 2.10 | Second-order theory |
| 113 | 2.10.1 | Josef Melan's contribution |
| 114 | 2.10.2 | Suspension bridges become stiffer |
| 115 | 2.10.3 | Arch bridges become more flexible |
| 115 | 2.10.4 | The differential equation for laterally loaded struts and ties |
| 116 | 2.10.5 | The integration of second-order theory into the displacement method |
| 117 | 2.10.6 | Why do we need fictitious forces? |
| 120 | 2.11 | Ultimate load method |
| 121 | 2.11.1 | First approaches |
| 123 | 2.11.2 | Foundation of the ultimate load method |
| 123 | 2.11.2.1 | Josef Fritsche |
| 124 | 2.11.2.2 | Karl Girkmann |
| 126 | 2.11.2.3 | Other authors |
| 127 | 2.11.3 | The paradox of the plastic hinge method |
| 130 | 2.11.4 | The establishment of the ultimate load method |
| 130 | 2.11.4.1 | Sir John Fleetwood Baker |
| 130 | 2.11.4.2 | Excursion: a sample calculation |
| 133 | 2.11.4.3 | Calculating deformations |
| 133 | 2.11.4.4 | The Anglo-American school of ultimate load theory |
| 135 | 2.11.4.5 | Controversies surrounding the ultimate load method |
| 137 | 2.12 | Structural law – Static law – Formation law |
| 137 | 2.12.1 | The five Platonic bodies |
| 139 | 2.12.2 | Beauty and law |
| 141 | 2.12.2.1 | Structural law |
| 142 | 2.12.2.2 | Static law |
| 142 | 2.12.2.3 | Formation law |
| 144 | 3 | The first fundamental engineering science disciplines: theory of structures and applied mechanics |
| 145 | 3.1 | What is engineering science? |
| 146 | 3.1.1 | First approaches |
| 148 | 3.1.2 | Raising the status of the engineering sciences through philosophical discourse |
| 150 | 3.1.2.1 | The contribution of systems theory |

| | | |
|-----|----------|--|
| 152 | 3.1.2.2 | The contribution of Marxism |
| 154 | 3.1.2.3 | Engineering sciences theory |
| 157 | 3.1.3 | Engineering and the engineering sciences |
| 161 | 3.2 | Subsuming the encyclopaedic in the system of classical engineering sciences: five case studies from applied mechanics and theory of structures |
| 162 | 3.2.1 | On the topicality of the encyclopaedic |
| 165 | 3.2.2 | Franz Joseph Ritter von Gerstner's contribution to the mathematisation of construction theories |
| 165 | 3.2.2.1 | Gerstner's definition of the object of applied mechanics |
| 168 | 3.2.2.2 | The strength of iron |
| 171 | 3.2.2.3 | The theory and practice of suspension bridges in <i>Handbuch der Mechanik</i> |
| 174 | 3.2.3 | Weisbach's encyclopaedia of applied mechanics |
| 174 | 3.2.3.1 | The <i>Lehrbuch</i> |
| 177 | 3.2.3.2 | The invention of the engineering manual |
| 179 | 3.2.3.3 | The journal |
| 180 | 3.2.3.4 | Strength of materials in Weisbach's <i>Lehrbuch</i> |
| 182 | 3.2.4 | Rankine's <i>Manuals</i> , or the harmony between theory and practice |
| 182 | 3.2.4.1 | Rankine's <i>Manual of Applied Mechanics</i> |
| 185 | 3.2.4.2 | Rankine's <i>Manual of Civil Engineering</i> |
| 186 | 3.2.5 | Föppl's <i>Vorlesungen über technische Mechanik</i> |
| 186 | 3.2.5.1 | The origin and goal of mechanics |
| 188 | 3.2.5.2 | The structure of the <i>Vorlesungen</i> |
| 189 | 3.2.5.3 | The most important applied mechanics textbooks in German |
| 190 | 3.2.6 | The <i>Handbuch der Ingenieurwissenschaften</i> as an encyclopaedia of classical civil engineering theory |
| 192 | 3.2.6.1 | Iron beam bridges |
| 193 | 3.2.6.2 | Iron arch and suspension bridges |
| 196 | 4 | From masonry arch to elastic arch |
| 199 | 4.1 | The arch allegory |
| 200 | 4.2 | The geometrical thinking behind the theory of masonry arch bridges |
| 200 | 4.2.1 | The Ponte S. Trinità in Florence |
| 203 | 4.2.1.1 | Galileo and Guidobaldo del Monte |
| 205 | 4.2.1.2 | Hypotheses |
| 205 | 4.2.2 | Establishing the new thinking in bridge-building practice using the example of Nuremberg's Fleisch Bridge |
| 206 | 4.2.2.1 | Designs for the building of the Fleisch Bridge |
| 207 | 4.2.2.2 | Designs and considerations concerning the centering |
| 208 | 4.2.2.3 | The loadbearing behaviour of the Fleisch Bridge |
| 211 | 4.3 | From wedge to masonry arch, or the addition theorem of wedge theory |
| 212 | 4.3.1 | Between mechanics and architecture: masonry arch theory at the Académie Royale d'Architecture de Paris (1687–1718) |
| 212 | 4.3.2 | La Hire and Bélidor |
| 214 | 4.3.3 | Epigones |
| 215 | 4.4 | From the analysis of masonry arch collapse mechanisms to voussoir rotation theory |
| 216 | 4.4.1 | Baldi |

| | | |
|-----|----------|---|
| 217 | 4.4.2 | Fabri |
| 218 | 4.4.3 | La Hire |
| 219 | 4.4.4 | Couplet |
| 221 | 4.4.5 | Bridge-building – empiricism still reigns |
| 222 | 4.4.6 | Coulomb’s voussoir rotation theory |
| 223 | 4.4.7 | Monasterio’s <i>Nueva Teórica</i> |
| 225 | 4.5 | The line of thrust theory |
| 225 | 4.5.1 | Prelude |
| 228 | 4.5.2 | Gerstner |
| 230 | 4.5.3 | The search for the true line of thrust |
| 232 | 4.6 | The breakthrough for elastic theory |
| 232 | 4.6.1 | The dualism of masonry arch and elastic arch theory under Navier |
| 233 | 4.6.2 | Two steps forwards, one back |
| 234 | 4.6.3 | From Poncelet to Winkler |
| 239 | 4.6.4 | A step back |
| 240 | 4.6.5 | The masonry arch is nothing, the elastic arch is everything – the triumph of elastic arch theory over masonry arch theory |
| 241 | 4.6.5.1 | <i>Grandes Voûtes</i> |
| 244 | 4.6.5.2 | Doubts |
| 245 | 4.6.5.3 | Tests on models |
| 247 | 4.7 | Ultimate load theory for masonry arches |
| 248 | 4.7.1 | Of cracks and the true line of thrust in the masonry arch |
| 250 | 4.7.2 | Masonry arch failures |
| 250 | 4.7.3 | The maximum load principles of the ultimate load theory for masonry arches |
| 251 | 4.7.4 | The safety of masonry arches |
| 252 | 4.7.5 | Analysis of masonry arch bridges |
| 256 | 4.7.6 | Heyman extends masonry arch theory |
| 258 | 4.8 | The finite element method |
| 262 | 4.9 | The studies of Holzer |
| 264 | 4.10 | On the epistemological status of masonry arch theories |
| 264 | 4.10.1 | Wedge theory |
| 265 | 4.10.2 | Collapse mechanism analysis and voussoir rotation theory |
| 266 | 4.10.3 | Line of thrust theory and elastic theory for masonry arches |
| 267 | 4.10.4 | Ultimate load theory for masonry arches as an object in historical theory of structures |
| 268 | 4.10.5 | The finite element analysis of masonry arches |
| 270 | 5 | The history of earth pressure theory |
| 272 | 5.1 | Retaining walls for fortifications |
| 275 | 5.2 | Earth pressure theory as an object of military engineering |
| 276 | 5.2.1 | In the beginning there was the inclined plane |
| 277 | 5.2.1.1 | Bullet |
| 278 | 5.2.1.2 | Gautier |
| 278 | 5.2.1.3 | Couplet |
| 279 | 5.2.1.4 | Further approaches |
| 280 | 5.2.1.5 | Friction reduces earth pressure |

| | | |
|-----|---------|--|
| 283 | 5.2.2 | From inclined plane to wedge theory |
| 286 | 5.2.3 | Charles Augustin Coulomb |
| 287 | 5.2.3.1 | Manifestations of adhesion |
| 288 | 5.2.3.2 | Failure behaviour of masonry piers |
| 289 | 5.2.3.3 | The transition to earth pressure theory |
| 290 | 5.2.3.4 | Active earth pressure |
| 294 | 5.2.3.5 | Passive earth pressure |
| 294 | 5.2.3.6 | Design |
| 295 | 5.2.4 | A magazine for engineering officers |
| 297 | 5.3 | Modifications to Coulomb earth pressure theory |
| 297 | 5.3.1 | The trigonometrisation of earth pressure theory |
| 297 | 5.3.1.1 | Prony |
| 298 | 5.3.1.2 | Mayniel |
| 299 | 5.3.1.3 | Français, Audoy and Navier |
| 301 | 5.3.1.4 | Martony de Köszezh |
| 303 | 5.3.2 | The geometric way |
| 304 | 5.3.2.1 | Jean-Victor Poncelet |
| 305 | 5.3.2.2 | Hermann Scheffler's criticism of Poncelet |
| 306 | 5.3.2.3 | Karl Culmann |
| 308 | 5.3.2.4 | Georg Rebhann |
| 310 | 5.3.2.5 | Compelling contradictions |
| 311 | 5.4 | The contribution of continuum mechanics |
| 313 | 5.4.1 | The hydrostatic earth pressure model |
| 314 | 5.4.2 | The new earth pressure theory |
| 316 | 5.4.2.1 | Carl Holtzmann |
| 316 | 5.4.2.2 | Rankine's stroke of genius |
| 317 | 5.4.2.3 | Emil Winkler |
| 319 | 5.4.2.4 | Otto Mohr |
| 321 | 5.5 | Earth pressure theory from 1875 to 1900 |
| 322 | 5.5.1 | Coulomb or Rankine? |
| 323 | 5.5.2 | Earth pressure theory in the form of masonry arch theory |
| 325 | 5.5.3 | Earth pressure theory <i>à la française</i> |
| 328 | 5.5.4 | Kötter's mathematical earth pressure theory |
| 331 | 5.6 | Experimental earth pressure research |
| 331 | 5.6.1 | The precursors of experimental earth pressure research |
| 332 | 5.6.1.1 | Cramer |
| 332 | 5.6.1.2 | Baker |
| 333 | 5.6.1.3 | Donath and Engels |
| 334 | 5.6.2 | A great moment in subsoil research |
| 336 | 5.6.3 | Earth pressure tests at the testing institute for the statics of structures at Berlin Technical University |
| 339 | 5.6.4 | The merry-go-round of discussions of errors |
| 341 | 5.6.5 | The Swedish school of earthworks |
| 343 | 5.6.6 | The emergence of soil mechanics |
| 344 | 5.6.6.1 | Three lines of development |
| 345 | 5.6.6.2 | The disciplinary configuration of soil mechanics |
| 345 | 5.6.6.3 | The contours of phenomenological earth pressure theory |

| | | |
|-----|----------|--|
| 348 | 5.7 | Earth pressure theory in the discipline-formation period of geotechnical engineering |
| 351 | 5.7.1 | Terzaghi |
| 352 | 5.7.2 | Rendulic |
| 352 | 5.7.3 | Ohde |
| 354 | 5.7.4 | Errors and confusion |
| 355 | 5.7.5 | A hasty reaction in print |
| 356 | 5.7.6 | Foundations + soil mechanics = geotechnical engineering |
| 356 | 5.7.6.1 | The civil engineer as soldier |
| 358 | 5.7.6.2 | Addendum |
| 360 | 5.8 | Earth pressure theory in the consolidation period of geotechnical engineering |
| 360 | 5.8.1 | New subdisciplines in geotechnical engineering |
| 361 | 5.8.2 | Determining earth pressure in practical theory of structures |
| 362 | 5.8.2.1 | The modified Culmann <i>E</i> line |
| 363 | 5.8.2.2 | New findings regarding passive earth pressure |
| 365 | 5.9 | Earth pressure theory in the integration period of geotechnical engineering |
| 366 | 5.9.1 | Computer-assisted earth pressure calculations |
| 367 | 5.9.2 | Geotechnical continuum models |
| 371 | 5.9.3 | The art of estimating |
| 373 | 5.9.4 | The history of geotechnical engineering as an object of construction history |
| 376 | 6 | The beginnings of a theory of structures |
| 378 | 6.1 | What is the theory of strength of materials? |
| 381 | 6.2 | On the state of development of theory of structures and strength of materials in the Renaissance |
| 387 | 6.3 | Galileo's Dialogue |
| 387 | 6.3.1 | First day |
| 390 | 6.3.2 | Second day |
| 396 | 6.4 | Developments in strength of materials up to 1750 |
| 404 | 6.5 | Civil engineering at the close of the 18th century |
| 405 | 6.5.1 | The completion of beam theory |
| 407 | 6.5.2 | Franz Joseph Ritter von Gerstner |
| 411 | 6.5.3 | Introduction to structural engineering |
| 412 | 6.5.3.1 | Gerstner's analysis and synthesis of loadbearing systems |
| 416 | 6.5.3.2 | Gerstner's method of structural design |
| 417 | 6.5.3.3 | <i>Einleitung in die statische Baukunst</i> as a textbook for analysis |
| 417 | 6.5.4 | Four comments on the significance of Gerstner's <i>Einleitung in die statische Baukunst</i> for theory of structures |
| 418 | 6.6 | The formation of a theory of structures: Eytelwein and Navier |
| 419 | 6.6.1 | Navier |
| 422 | 6.6.2 | Eytelwein |
| 424 | 6.6.3 | The analysis of the continuous beam according to Eytelwein and Navier |

| | | |
|-----|----------|--|
| 425 | 6.6.3.1 | The continuous beam in Eytelwein's <i>Statik fester Körper</i> |
| 429 | 6.6.3.2 | The continuous beam in Navier's <i>Résumé des Leçons</i> |
| 432 | 6.7 | Adoption of Navier's analysis of the continuous beam |
| 436 | 7 | The discipline-formation period of theory of structures |
| 438 | 7.1 | Clapeyron's contribution to the formation of the classical engineering sciences |
| 438 | 7.1.1 | <i>Les polytechniciens</i> : the fascinating revolutionary élan in post-revolution France |
| 440 | 7.1.2 | Clapeyron and Lamé in St. Petersburg (1820–1831) |
| 443 | 7.1.3 | Clapeyron's formulation of the energy doctrine of the classical engineering sciences |
| 445 | 7.1.4 | Bridge-building and the theorem of three moments |
| 448 | 7.2 | The completion of the practical beam theory |
| 451 | 7.3 | From graphical statics to graphical analysis |
| 452 | 7.3.1 | The founding of graphical statics by Culmann |
| 454 | 7.3.2 | Two graphical integration machines |
| 455 | 7.3.3 | Rankine, Maxwell, Cremona and Bow |
| 457 | 7.3.4 | Differences between graphical statics and graphical analysis |
| 459 | 7.3.5 | The breakthrough for graphical analysis |
| 460 | 7.3.5.1 | Graphical analysis of masonry vaults and domes |
| 462 | 7.3.5.2 | Graphical analysis in engineering works |
| 465 | 7.4 | The classical phase of theory of structures |
| 465 | 7.4.1 | Winkler's contribution |
| 468 | 7.4.1.1 | The elastic theory foundation to theory of structures |
| 471 | 7.4.1.2 | The theory of the elastic arch as a foundation for bridge-building |
| 476 | 7.4.2 | The beginnings of the force method |
| 476 | 7.4.2.1 | Contributions to the theory of statically indeterminate trussed frameworks |
| 481 | 7.4.2.2 | From the trussed framework theory to the general theory of trusses |
| 491 | 7.4.3 | Loadbearing structure as kinematic machine |
| 492 | 7.4.3.1 | Trussed framework as machine |
| 493 | 7.4.3.2 | The theoretical kinematics of Reuleaux and the Dresden school of kinematics |
| 495 | 7.4.3.3 | Kinematic or energy doctrine in theory of structures? |
| 499 | 7.4.3.4 | The Pyrrhic victory of the energy doctrine in theory of structures |
| 500 | 7.5 | Theory of structures at the transition from the discipline-formation to the consolidation period |
| 500 | 7.5.1 | Castigliano |
| 504 | 7.5.2 | The fundamentals of classical theory of structures |
| 508 | 7.5.3 | Resumption of the dispute about the fundamentals of classical theory of structures |
| 508 | 7.5.3.1 | The cause |
| 509 | 7.5.3.2 | The dispute between the 'seconds' |
| 510 | 7.5.3.3 | The dispute surrounding the validity of the theorems of Castigliano |
| 516 | 7.5.4 | The validity of Castigliano's theorems |

| | | |
|-----|----------|---|
| 517 | 7.6 | Lord Rayleigh's <i>The Theory of Sound</i> and Kirpitchenov's fundamentals of classical theory of structures |
| 517 | 7.6.1 | Rayleigh coefficient and Ritz coefficient |
| 520 | 7.6.2 | Kirpitchenov's congenial adaptation |
| 522 | 7.7 | The Berlin school of theory of structures |
| 523 | 7.7.1 | The notion of the scientific school |
| 524 | 7.7.2 | The completion of classical theory of structures by Müller-Breslau |
| 526 | 7.7.3 | Classical theory of structures usurps engineering design |
| 530 | 7.7.4 | Müller-Breslau's students |
| 531 | 7.7.4.1 | August Hertwig |
| 534 | 7.7.4.2 | August Hertwig's successors |
| 538 | 8 | From construction with iron to modern structural steelwork |
| 541 | 8.1 | Torsion theory in iron construction and theory of structures from 1850 to 1900 |
| 541 | 8.1.1 | Saint-Venant's torsion theory |
| 545 | 8.1.2 | The torsion problem in Weisbach's <i>Principles</i> |
| 547 | 8.1.3 | Bach's torsion tests |
| 550 | 8.1.4 | The adoption of torsion theory in classical theory of structures |
| 553 | 8.2 | Crane-building at the focus of mechanical and electrical engineering, steel construction and theory of structures |
| 553 | 8.2.1 | Rudolph Bredt – known yet unknown |
| 554 | 8.2.2 | The Ludwig Stuckenholz company in Wetter a. d. Ruhr |
| 555 | 8.2.2.1 | Bredt's rise to become the master of crane-building |
| 559 | 8.2.2.2 | Crane types of the Ludwig Stuckenholz company |
| 564 | 8.2.3 | Bredt's scientific-technical publications |
| 565 | 8.2.3.1 | Bredt's testing machine |
| 566 | 8.2.3.2 | The principle of separating the functions in crane-building |
| 567 | 8.2.3.3 | Crane hooks |
| 567 | 8.2.3.4 | Struts |
| 567 | 8.2.3.5 | Foundation anchors |
| 568 | 8.2.3.6 | Pressure cylinders |
| 568 | 8.2.3.7 | Curved bars |
| 568 | 8.2.3.8 | Elastic theory |
| 569 | 8.2.3.9 | The teaching of engineers |
| 570 | 8.2.3.10 | Torsion theory |
| 571 | 8.2.4 | Heavy engineering adopts classical theory of structures |
| 575 | 8.3 | Torsion theory in the consolidation period of theory of structures (1900–1950) |
| 575 | 8.3.1 | The introduction of an engineering science concept: the torsion constant |
| 577 | 8.3.2 | The discovery of the shear centre |
| 578 | 8.3.2.1 | Carl Bach |
| 579 | 8.3.2.2 | Louis Potterat |
| 579 | 8.3.2.3 | Adolf Eggenschwyler |
| 580 | 8.3.2.4 | Robert Maillart |
| 582 | 8.3.2.5 | Rearguard actions in the debate surrounding the shear centre |

| | | |
|-----|----------|--|
| 582 | 8.3.3 | Torsion theory in structural steelwork from 1925 to 1950 |
| 585 | 8.3.4 | Summary |
| 585 | 8.4 | Searching for the true buckling theory in steel construction |
| 585 | 8.4.1 | The buckling tests of the DStV |
| 587 | 8.4.1.1 | The world's largest testing machine |
| 588 | 8.4.1.2 | The perfect buckling theory on the basis of elastic theory |
| 590 | 8.4.2 | German State Railways and the joint technical-scientific work in structural steelwork |
| 590 | 8.4.2.1 | Standardising the codes of practice for structural steelwork |
| 592 | 8.4.2.2 | The founding of the German Committee for Structural Steelwork (DAST) |
| 593 | 8.4.3 | Excursion: the "Olympic Games" for structural engineering |
| 595 | 8.4.4 | A paradigm change in buckling theory |
| 596 | 8.4.5 | The standardisation of the new buckling theory in the German stability standard DIN 4114 |
| 599 | 8.5 | Steelwork and steelwork science from 1925 to 1975 |
| 600 | 8.5.1 | From the one-dimensional to the two-dimensional structure |
| 600 | 8.5.1.1 | The theory of the effective width |
| 603 | 8.5.1.2 | Constructional innovations in German bridge-building during the 1930s |
| 606 | 8.5.1.3 | The theory of the beam grid |
| 608 | 8.5.1.4 | The orthotropic plate as a patent |
| 609 | 8.5.1.5 | Structural steelwork borrows from reinforced concrete: Huber's plate theory |
| 612 | 8.5.1.6 | The Guyon-Massonnet method |
| 613 | 8.5.1.7 | The theory dynamic in steelwork science in the 1950s and 1960s |
| 615 | 8.5.2 | The rise of steel-concrete composite construction |
| 616 | 8.5.2.1 | Composite columns |
| 617 | 8.5.2.2 | Composite beams |
| 621 | 8.5.2.3 | Composite bridges |
| 628 | 8.5.3 | Lightweight steel construction |
| 632 | 8.5.4 | Steel and glass – best friends |
| 637 | 8.6 | Eccentric orbits – the disappearance of the centre |
| 640 | 9 | Member analysis conquers the third dimension: the spatial framework |
| 641 | 9.1 | The emergence of the theory of spatial frameworks |
| 644 | 9.1.1 | The original dome to the Reichstag (German parliament building) |
| 645 | 9.1.2 | Foundation of the theory of spatial frameworks by August Föppl |
| 649 | 9.1.3 | Integration of spatial framework theory into classical theory of structures |
| 652 | 9.2 | Spatial frameworks in an age of technical reproducibility |
| 653 | 9.2.1 | Alexander Graham Bell |
| 654 | 9.2.2 | Vladimir Grigorievich Shukhov |
| 655 | 9.2.3 | Walther Bauersfeld and Franz Dischinger |
| 656 | 9.2.4 | Richard Buckminster Fuller |
| 657 | 9.2.5 | Max Mengerinhausen |

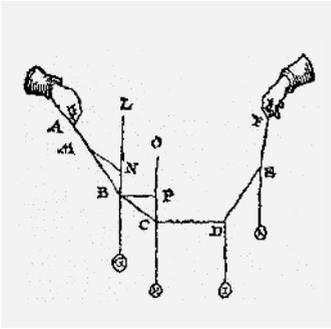
| | | |
|-----|-----------|---|
| 658 | 9.3 | Dialectic synthesis of individual structural composition and large-scale production |
| 659 | 9.3.1 | The MERO system and the composition law for spatial frameworks |
| 661 | 9.3.2 | Spatial frameworks and computers |
| 664 | 10 | Reinforced concrete's influence on theory of structures |
| 666 | 10.1 | The first design methods in reinforced concrete construction |
| 666 | 10.1.1 | The beginnings of reinforced concrete construction |
| 668 | 10.1.2 | From the German Monier patent to the <i>Monier-Broschüre</i> |
| 671 | 10.1.3 | The <i>Monier-Broschüre</i> |
| 672 | 10.1.3.1 | The new type of structural-constructional quality offered by the Monier system |
| 673 | 10.1.3.2 | The applications of the Monier system |
| 675 | 10.1.3.3 | The engineering science principles of the Monier system |
| 679 | 10.2 | Reinforced concrete revolutionises the building industry |
| 681 | 10.2.1 | The fate of the Monier system |
| 682 | 10.2.2 | The end of the system period: steel + concrete = reinforced concrete |
| 684 | 10.2.2.1 | The Napoleon of reinforced concrete: François Hennebique |
| 686 | 10.2.2.2 | The founding father of rationalism in reinforced concrete: Paul Christophe |
| 691 | 10.2.2.3 | The completion of the triad |
| 696 | 10.3 | Theory of structures and reinforced concrete |
| 697 | 10.3.1 | New types of loadbearing structure in reinforced concrete |
| 698 | 10.3.1.1 | Reinforced concrete gains emancipation from structural steelwork: the rigid frame |
| 702 | 10.3.1.2 | Reinforced concrete takes its first steps into the second dimension: out-of-plane-loaded structures |
| 717 | 10.3.1.3 | The first synthesis |
| 719 | 10.3.2 | The structural-constructional self-discovery of reinforced concrete |
| 720 | 10.3.2.1 | In-plane-loaded elements and folded plates |
| 722 | 10.3.2.2 | Reinforced concrete shells |
| 757 | 10.3.2.3 | The second synthesis |
| 760 | 10.3.2.4 | Of the power of formalised theory |
| 762 | 10.4 | Prestressed concrete: "Une révolution dans l'art de bâtir" (Freysinnet) |
| 763 | 10.4.1 | Leonhardt's <i>Prestressed Concrete. Design and Construction</i> |
| 766 | 10.4.2 | The first prestressed concrete standard |
| 767 | 10.4.3 | Prestressed concrete standards in the GDR |
| 769 | 10.4.4 | The unstoppable rise of prestressed concrete reflected in <i>Beton- und Stahlbetonbau</i> |
| 770 | 10.5 | Paradigm change in reinforced concrete design in the Federal Republic of Germany, too |
| 772 | 10.6 | Revealing the invisible: reinforced concrete design with truss models |
| 772 | 10.6.1 | The trussed framework model of François Hennebique |
| 773 | 10.6.2 | The trussed framework model of Emil Mörsch |
| 775 | 10.6.3 | A picture is worth 1,000 words: stress patterns for plane plate and shell structures |

| | | |
|-----|-----------|--|
| 777 | 10.6.4 | The concept of the truss model: steps towards holistic design in reinforced concrete |
| 780 | 11 | The consolidation period of theory of structures |
| 781 | 11.1 | The relationship between text, image and symbol in theory of structures |
| 783 | 11.1.1 | The historical stages in the idea of formalisation |
| 790 | 11.1.2 | The structural engineer – a manipulator of symbols? |
| 791 | 11.2 | The development of the displacement method |
| 792 | 11.2.1 | The contribution of the mathematical elastic theory |
| 793 | 11.2.1.1 | Elimination of stresses or displacements? That is the question. |
| 794 | 11.2.1.2 | An element from the ideal artefacts of mathematical elastic theory: the elastic truss system |
| 795 | 11.2.2 | From pin-jointed trussed framework to rigid-jointed frame |
| 795 | 11.2.2.1 | A real engineering artefact: the iron trussed framework with riveted joints |
| 797 | 11.2.2.2 | The theory of secondary stresses |
| 799 | 11.2.3 | From trussed framework to rigid frame |
| 800 | 11.2.3.1 | Thinking in deformations |
| 802 | 11.2.3.2 | The Vierendeel girder |
| 803 | 11.2.4 | The displacement method gains emancipation from trussed framework theory |
| 805 | 11.2.4.1 | Axel Bendixsen |
| 806 | 11.2.4.2 | George Alfred Maney |
| 806 | 11.2.4.3 | Willy Gehler |
| 807 | 11.2.4.4 | Asger Ostenfeld |
| 808 | 11.2.4.5 | Peter L. Pasternak |
| 808 | 11.2.4.6 | Ludwig Mann |
| 809 | 11.2.5 | The displacement method during the invention phase of theory of structures |
| 810 | 11.3 | The rationalisation movement in theory of structures |
| 811 | 11.3.1 | The prescriptive use of symbols in theory of structures |
| 814 | 11.3.2 | Rationalisation of statically indeterminate calculations |
| 815 | 11.3.2.1 | Statically indeterminate main systems |
| 816 | 11.3.2.2 | Orthogonalisation methods |
| 817 | 11.3.2.3 | Specific methods from the theory of sets of linear equations |
| 818 | 11.3.2.4 | Structural iteration methods |
| 821 | 11.3.3 | The dual nature of theory of structures |
| 824 | 11.4 | Konrad Zuse and the automation of structural calculations |
| 824 | 11.4.1 | Schematisation of statically indeterminate calculations |
| 826 | 11.4.1.1 | Schematic calculation procedure |
| 829 | 11.4.1.2 | The first step to the computing plan |
| 832 | 11.4.2 | The “engineer’s calculating machine” |
| 834 | 11.5 | Matrix formulation |
| 834 | 11.5.1 | Matrix formulation in mathematics and theoretical physics |
| 835 | 11.5.2 | Tensor and matrix algebra in the fundamental engineering science disciplines |
| 838 | 11.5.3 | The integration of matrix formulation into engineering mathematics |
| 841 | 11.5.4 | A structural analysis matrix method: the carry-over method |

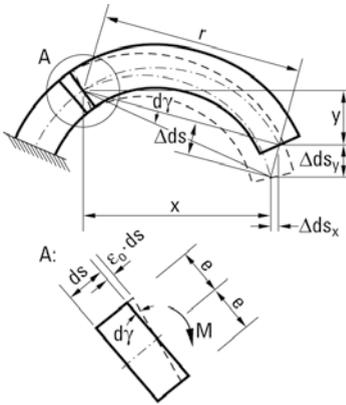
| | | |
|-----|-----------|--|
| 846 | 12 | The development and establishment of computational statics |
| 847 | 12.1 | “The computer shapes the theory” (Argyris) – the historical roots of the finite element method |
| 850 | 12.1.1 | Truss models for elastic continua |
| 850 | 12.1.1.1 | Kirsch’s space truss model |
| 851 | 12.1.1.2 | Trussed framework models for elastic plates |
| 853 | 12.1.1.3 | The origin of the gridwork method |
| 855 | 12.1.1.4 | First computer-aided structural analyses in the automotive industry |
| 859 | 12.1.2 | Modularisation and discretisation of aircraft structures |
| 859 | 12.1.2.1 | From lattice box girder to cell tube and shear field layout |
| 866 | 12.1.2.2 | High-speed aerodynamics, discretisation of the cell tube and matrix theory |
| 869 | 12.2 | The matrix algebra reformulation of structural mechanics |
| 870 | 12.2.1 | The founding of modern structural mechanics |
| 873 | 12.2.2 | The first steps towards computational statics in Europe |
| 873 | 12.2.2.1 | Switzerland |
| 875 | 12.2.2.2 | United Kingdom |
| 877 | 12.2.2.3 | Federal Republic of Germany |
| 880 | 12.3 | FEM – formation of a general technology of engineering science theory |
| 881 | 12.3.1 | The classical publication of a non-classical method |
| 884 | 12.3.2 | The heuristic potential of FEM: the direct stiffness method |
| 887 | 12.4 | The founding of FEM through variational principles |
| 888 | 12.4.1 | The variational principle of Dirichlet and Green |
| 888 | 12.4.1.1 | A simple example: the axially loaded elastic extensible bar |
| 890 | 12.4.1.2 | The Göttingen school around Felix Klein |
| 891 | 12.4.2 | The first stage of the synthesis: the canonic variational principle of Hellinger and Prange |
| 892 | 12.4.2.1 | Prange’s habilitation thesis |
| 895 | 12.4.2.2 | In the Hades of amnesia |
| 896 | 12.4.2.3 | First steps in recollection |
| 896 | 12.4.2.4 | Eric Reissner’s contribution |
| 898 | 12.4.3 | The second stage of the synthesis: the variational principle of Fraeijs de Veubeke, Hu and Washizu |
| 901 | 12.4.4 | The variational formulation of FEM |
| 904 | 12.4.5 | A break with symmetry with serious consequences |
| 905 | 12.5 | Back to the roots |
| 907 | 12.5.1 | Priority for mathematical reasoning |
| 908 | 12.5.2 | Influence functions |
| 909 | 12.5.3 | Influence functions and FEM – an example |
| 910 | 12.5.4 | Practical benefits of influence functions |
| 910 | 12.5.5 | The fundamentals of theory of structures |
| 911 | 12.6 | Computational mechanics |
| 916 | 13 | Thirteen scientific controversies in mechanics and theory of structures |
| 917 | 13.1 | The scientific controversy |
| 917 | 13.2 | Thirteen disputes |
| 917 | 13.2.1 | Galileo’s <i>Dialogo</i> |
| 918 | 13.2.2 | Galileo’s <i>Discorsi</i> |

| | | |
|------|-----------|--|
| 919 | 13.2.3 | The philosophical dispute about the true measure of force |
| 920 | 13.2.4 | The dispute about the principle of least action |
| 921 | 13.2.5 | The dome of St. Peter's in the dispute between theorists and practitioners |
| 923 | 13.2.6 | Discontinuum or continuum? |
| 924 | 13.2.7 | Graphical statics vs. graphical analysis, or the defence of pure theory |
| 925 | 13.2.8 | Animosity creates two schools: Mohr vs. Müller-Breslau |
| 926 | 13.2.9 | The war of positions |
| 927 | 13.2.10 | Until death do us part: Fillunger vs. Terzaghi |
| 929 | 13.2.11 | "In principle, yes ...": the dispute about principles |
| 931 | 13.2.12 | Elastic or plastic? That is the question. |
| 932 | 13.2.13 | The importance of the classical earth pressure theory |
| 933 | 13.3 | Résumé |
| 934 | 14 | Perspectives for a historical theory of structures |
| 936 | 14.1 | Theory of structures and aesthetics |
| 936 | 14.1.1 | The schism of architecture |
| 937 | 14.1.2 | Beauty and utility in architecture – a utopia? |
| 941 | 14.1.3 | Alfred Gotthold Meyer's <i>Eisenbauten. Ihre Geschichte und Ästhetik</i> |
| 945 | 14.1.4 | The aesthetics in the dialectic between building and calculation |
| 948 | 14.2 | Historical engineering science – historical theory of structures |
| 950 | 14.2.1 | Saint-Venant's historical elastic theory |
| 952 | 14.2.2 | Historical masonry arch theory |
| 952 | 14.2.3 | Historico-genetic teaching of theory of structures |
| 954 | 14.2.3.1 | The historico-logical longitudinal analysis |
| 954 | 14.2.3.2 | The historico-logical cross-sectional analysis |
| 955 | 14.2.3.3 | The historico-logical comparison |
| 955 | 14.2.3.4 | Content, aims, means and characteristics of the historico-genetic teaching of theory of structures |
| 958 | 14.2.4 | Computer-assisted graphical analysis |
| 962 | 15 | Brief biographies of 260 protagonists of theory of structures |
| 1090 | | Bibliography |
| 1184 | | Name index |
| 1196 | | Subject index |

Chapter 4



From masonry arch to elastic arch



The masonry arch is still one of the mysteries of architecture. Anybody who looks into the history of theory of structures quickly encounters this puzzle, the solution to which has occupied countless numbers of scientists and engineers right up to the present day. Since completing his diploma at the Faculty of Theory of Structures at Berlin Technical University in 1981, the author can be counted as belonging to that group. Those studies introduced him to Jacques Heyman's work on the history of theory, which the latter developed into his masonry arch model based on ultimate load theory. A lecture given at the Faculty of Civil Engineering at Stuttgart University, instigated by Prof. Ekkehard Ramm, resulted in a work summarising the development of masonry arch theories since Leonardo da Vinci – and forms the crux of this chapter. Section 4.2.1 was written by Andreas Kahlow and section 4.2.2 by Holger Falter; new findings have found their way into both these sections, one example being the dissertation by Christiane Kaiser. The author would like to take this opportunity to thank Andreas Kahlow and Holger Falter for their kind permission to reproduce their work in this book. The excellent researches of Antonio Becchi, Federico Focé and Santiago Huerta contributed to the success of sections 4.3.1, 4.4.1, 4.4.7 and 4.7; friendships grew out of our many years of cooperation in the field of the history of construction. Numerous ideas resulted from the research of Stefan M. Holzer in the area of the structural assessment of arch structures. The author's dream of a theory of structures within the framework of a historical engineering science took shape through the works of the aforementioned researchers.

Jakob Grimm (1785–1863) and Wilhelm Grimm (1786–1859) describe the German noun *Bogen* (= bow, curve, arch) as “... that which is curved, is becoming curved, is rising in a curve” [Grimm, 1860, p. 91], the roots of which lie in the German verb *biegen* (= to bend). A bow (i. e. arch, from *arcus*, the Latin word for arc, bow) in the structural sense is consequently a concave loadbearing structure whose load-carrying mechanism is achieved by way of rigid building materials such as timber, steel and reinforced concrete. When loading such a curved loadbearing structure, a non-negligible part of the external work is converted into internal bending work. Therefore, in German the verb *biegen* not only constitutes the etymological foundation for the noun *Bogen*, but also characterises the curved loadbearing structure from the point of view of the load-carrying mechanism in a very visual and memorable way.

The genesis of the German noun *Gewölbe* (= vault, from *voluta*, the Latin word for roll, turn) is much more complex. Its roots are to be found in Roman stone buildings, as opposed to timber buildings, and in particular the Roman camera, i. e. initially the arched or vaulted ceiling or chamber: “Actually only the word for the curved ceiling, ... ‘camera’ gradually became the term for the whole room below the ceiling. And it is this shift in meaning, which is repeated similarly in ‘*Gewölbe*’, that leads to the majority of uses for which the latter is regarded as characteristic” [Grimm, 1973, p. 6646].

It is in the German building terminology of the 18th century that we first see the word *Gewölbe* being used in its two-dimensional meaning, whereupon the three-dimensional sense was quickly forgotten. The reason for this may well have been the masonry arch theories that began to surface in the century of the Enlightenment, which started the transitions from loadbearing structure to loadbearing system as a masonry arch model abstracted from the point of view of the loadbearing function – and therefore permitted a quantitative assessment of the load-carrying mechanism in the arch. The beam theory that began with Galileo acted as complement to this terminological refinement. In Zedler’s *Universal-Lexikon* dating from 1735, for example, *Gewölbe* is defined totally in the two-dimensional sense, “a curved stone ceiling” [Zedler, 1735, p.1393], and is differentiated from the suspended timber floor subjected to bending. In 1857 Ersch and Gruber expanded the definition on the basis of the two-dimensional term by mentioning, in addition to dressed stones and bricks, rubble stone material (with mortar joints) as a building material for vaults and arches [Ersch & Gruber, 1857, p.129]. This became apparent in the material homogenisation of the masonry arch structure that began around 1850 in France, which, in the shape of the plain and reinforced concrete structures of the final decades of the 19th century, paved the way – in the construction sense – for the transition from the theories linked with the materials of the loadbearing masonry arch to the elastic masonry arch theories of Saavedra (1860), Rankine (1862), Perrodil (1872, 1876, 1879, 1880 & 1882), Castigliano (1879), Winkler (1879/1880) and others, and from there to elastic arch theory. The logical nucleus of this historical process is

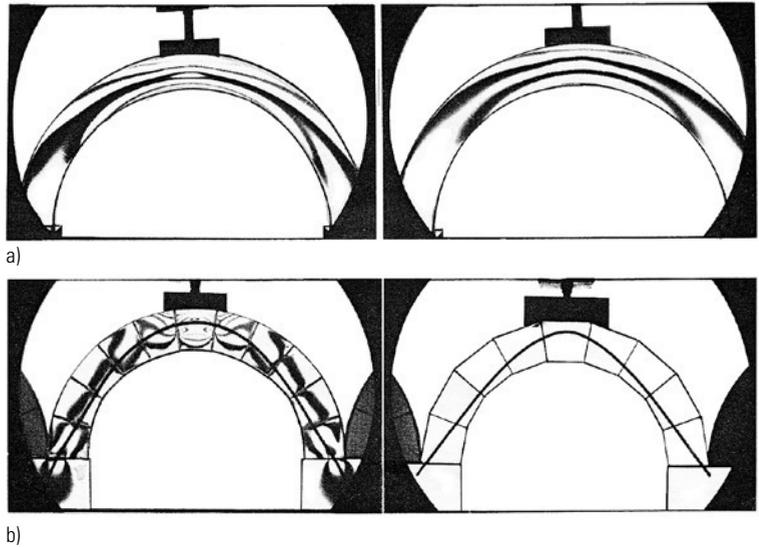


FIGURE 4-1
Photoelastic experiment carried out on a model subjected to a central point load: isochromatic lines of a) monolithic arch model and b) masonry arch model [Heinrich, 1979, pp. 37–38]

the transition from the loadbearing system to the structural system of the elastic arch, e.g. as a concave elastic bar fixed at the abutments. Another thread in elastic arch theory leads us back to the history of timber structures, which Holzer has pursued in two remarkable essays [Holzer, 2007 & 2010/2].

The German noun *Gewölbe* is still used to form compound designations for a number of arch structures, e.g. *Stahlgewölbe* and *Stahlbetongewölbe* [Badr, 1962, p. 43ff.] (steel and reinforced concrete arches respectively). This contradicts the view that such loadbearing structures work not only in compression, but also in bending as linear-elastic, concave continua. The photoelastic experiments of Bert Heinrich proved the conceptual difference between *Bogen* and *Gewölbe*. Whereas the parallel isochromatic lines in the homogeneous arch indicate high bending stresses (Fig. 4-1a), the loadbearing quality of the (inhomogeneous) masonry arch is characterised purely by the propagation of compression in the direction of the thrust line (Fig. 4-1b).

Summing up, the following definition is proposed: A concave loadbearing structure is a masonry arch when the provision of the loadbearing function is realised solely through rigid building materials with negligible tensile strength which are joined together. Weber has refined this definition and proposed one based on the two-dimensional concept of differential geometry [Weber, 1999, pp. 30–37].

The invention of the masonry arch is, like that of the wheel, impossible to date. In the Berlin Museum of Prehistory & Ancient History, visitors can admire a Mesopotamian burial chamber more than 5,000 years old which is in the form of a barrel vault with a span of a little over 1 m. “False and true arches as used over canals and crypts,” writes Ernst Heinrich, “could well date from about the same period even if the one is known to us from the Uruk age, the other from the Mesilim. Both remain ... in use until the time of the Seleucids” [Heinrich, 1957–1971, p. 339]. There

are without doubt various historico-logical chains of development that culminate in the masonry arch. It is not difficult to imagine that during the construction of a false or corbelled arch the upper stones may have fallen inwards and wedged themselves into an arch shape (Fig. 4-2a), or one or more wedges could have been inserted between two mutually supportive stone slabs to enable the use of shorter slabs (Fig. 4-2b). The same technical motive to reduce the length of a beam and hence increase the bending strength may have encouraged ancient builders to switch from the lintel to the flat arch (see [Huerta, 2012] for the history of the theory of the flat arch) and then to the arch (Fig. 4-2c).

More than 2,000 years certainly passed before the Etruscans' masonry arch with specially cut joints appeared. But the span of time from the first masonry arch theories of the late 17th century to elastic arch theory is less than 200 years. And the analysis of masonry arches based on the ultimate load method did not appear on the scene until the 1960s.

4.1 The arch allegory

Shortly before Christmas 2010, this author received a remarkable letter from Klaus Stiglat [Stiglat, 2010]. The writer of the letter steered the recipient's interest to the arch allegory of the poet Heinrich von Kleist (1777–1811) [Földényi, 1999, pp. 161–163].

According to Kleist himself, 16 November 1800 was the “most important day” of his life. As he wrote in a two-part letter to Wilhelmine von Zenge (1780–1852) dated 16 and 18 November 1800: “... in Würzburg, I went for a walk. ... When the sun went down, it seemed as though my happiness were sinking with it. I was walking back to the city, lost in my own thoughts, through an arched gate. Why, I asked myself, does this arch not collapse, since after all it has *no* support? It remains standing, I answered, *because all the stones tend to collapse at the same time* – and from this thought I derived an indescribably heartening consolation, which stayed by me right up to the decisive moment: I too would not collapse, even if all my support were removed!” (trans. by [Miller, 1982] cited in: [Madsen, 2016, p. 10]). Kleist drew a sketch of the arched gateway in Würzburg and sent it to his “Dear Wilhelmine” on 30 December 1800 (Fig. 4-3).

Kleist's sketch shows seven wedge-shaped stones with the keystone emphasised and a tie that resists the horizontal thrust of the arch. In the ninth scene of his play *Penthesilea* (1808), Prothoë says the following to Penthesilea: “... Stand, stand as does the vaulted arch stand firm, / Because each of its blocks inclines to fall!” (trans. by [Agee, 1998] cited in: [Allen, 2005/2007, pp. 25–26]).

In his letter, Klaus Stiglat comments on Kleist's arch allegory thus: “So stability and ‘statics’ can also be expressed in that way, too – lending humankind stability and ‘sanity’” [Stiglat, 2010].

Kleist's image of the lintel as support is more than just the essence of a private theory shared with Wilhelmine von Zenge, as Günter Blamberger writes [Blamberger, 2011, p. 66]. Instead, in the form of the gauged arch, it represents statics as a theory of equilibrium per se – yet announcing the lintel as support through the wedging together of the stones at the histori-

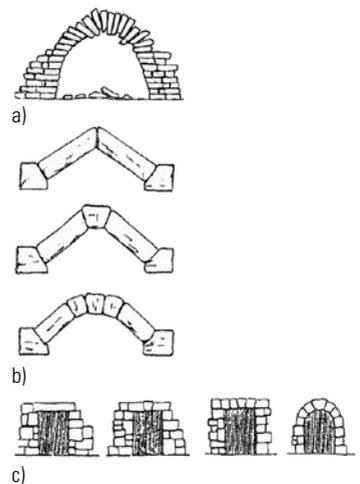


FIGURE 4-2
Historico-logical developments:
a) corbelled arch, b) three-hinge system,
and c) from lintel to masonry arch
(Heinrich, 1979, pp. 24–25)



FIGURE 4-3
Kleist's sketch of the arched gateway
in Würzburg [Blamberger, 2011, p. 66]

**The geometrical thinking
behind the theory of masonry
arch bridges**

co-logical transition from the false to the true arch (Fig. 4-2a) a completely new type of equilibrium configuration.

4.2

Whereas the large bridges of the late Renaissance demonstrated innovations primarily through the use of geometry, the application of the methods of statics in design remained the province of the Baroque.

More precise variation in possible design geometries, the centering, the foundations and the construction sequence, etc. was now feasible through the use of drawings, ever-better dimensional accuracy and precision in the designs. Using the examples of the Ponte S. Trinità in Florence and the Fleisch Bridge in Nuremberg, it will be shown how these new design approaches gradually became accepted in bridge-building.

During the first decades of the 18th century, bridge-building progressed via the intermediate stages of the first attempts to quantify this subject (La Hire, Couplet, Bélidor) to become the number one object of masonry arch theory. The idea of the thrust line became, indirectly, the focus of all deliberations: conceptual designs concerning the functional mechanism of bridges and intensive communication between experts advanced the formulation of bridge-building theories.

**The Ponte S. Trinità
in Florence**

4.2.1

The end of the 16th century marked the start of a new evolutionary era in the building of masonry arch bridges. The Renaissance initially took the structures and forms of construction of the Romans as its models. Owing to its rise/span ratio of 1:2, the semicircular arch permits only very restricted functionality and is therefore unsuitable for urban structures in particular. This functional disadvantage gave rise to new arch forms that were considerably shallower than the Roman arch.

Besides longer spans, the rise/span ratio also increased. The classical ratio was around 1:3, but in the case of the Ponte Vecchio (5 m rise, 32 m span) by Taddeo Gaddi (1300–1366), this increased during the late Middle Ages to 1:6.5. However, a new approach to design – and not just spans longer than those of the late Middle Ages – was the main aspect that signalled the leap in quality of the Renaissance compared with ancient

FIGURE 4-4
Ponte S. Trinità, photo taken prior to the bridge's destruction in the Second World War (photo: Gizdulich collection)



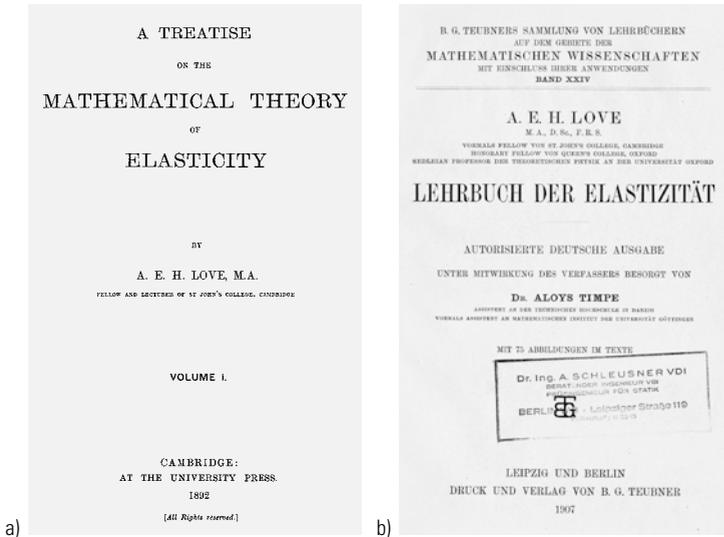


FIGURE 10-40
Title pages of a) the first volume [Love, 1892/1893], and b) the German translation of the collected edition [Love, 1907] of Love's *Treatise*

classical phase (1875–1900) and the accumulation phase (1900–1925) of theory of structures that engineers built the golden bridge from mathematical to analytical shell theory.

Practising engineers initially approached shell theory cautiously via the analysis of the simplest shell form, the single-curvature, fixed cylindrical shell; but the representatives of fundamental engineering science disciplines such as applied mechanics and theory of structures were no different (Fig. 10-41). Using this structural model, engineers attempted to size vessels of steel and, later, reinforced concrete – the works of E. Winkler (1860), F. Grashof (1878), G. A. Wayss (1887), V. G. Shukhov (1888) (see [Ramm, 1990]), P. Forchheimer (1894), R. Maillart (1903) (see [Schöne, 1999, 2011]), C. Runge (1904), Panetti (1906), H. Müller-Breslau (1908), H. Reissner (1908), K. Federhofer (1909, 1910), T. Pöschl and K. v. Terzaghi (1913) and A. and L. Föppl (1920) should be mentioned here. In 1923 V. Lewe summarised the methods for the structural calculation of liquid-retaining structures in a longer article for the *Handbuch für Eisenbetonbau* (reinforced concrete manual) [Lewe, 1923].

In his *Monier-Broschüre*, G. A. Wayss specifies an equation for determining the wall thickness $t(z)$ of a reinforced concrete water tank [Wayss, 1887, p. 34] which he derived from the boiler formula (eq. 8-35) (Fig. 10-41):

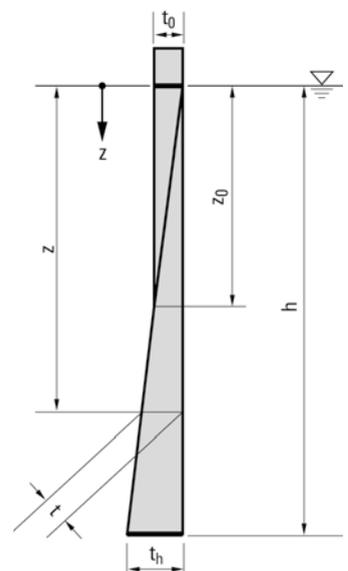
$$t(z) = t = r \cdot \frac{p_i}{\sigma_{\text{permiss}}} = r \cdot \frac{\gamma \cdot z}{\left[\sigma_{b,\text{permiss}} + \frac{1}{n} (\sigma_{s,\text{permiss}} - \sigma_{b,\text{permiss}}) \right]} \quad (10-55)$$

where:

- r internal radius of water tank
- $t(z)$ wall thickness
- $\gamma \cdot z$ hydrostatic pressure at depth z below the surface of the water
- $\sigma_{b,\text{permiss}}$ permissible tensile stress of concrete
- $\sigma_{s,\text{permiss}}$ permissible tensile stress of steel

Practice makes do: from tank formula to tank theory

FIGURE 10-41
On the design of a reinforced concrete cylindrical water tank with a partially linearly varying wall thickness after Wayss [Wayss, 1887, p. 34]



n ratio of concrete cross-sectional area A_b to steel cross-sectional area A_s (i. e. amount of reinforcement per unit length in z direction)

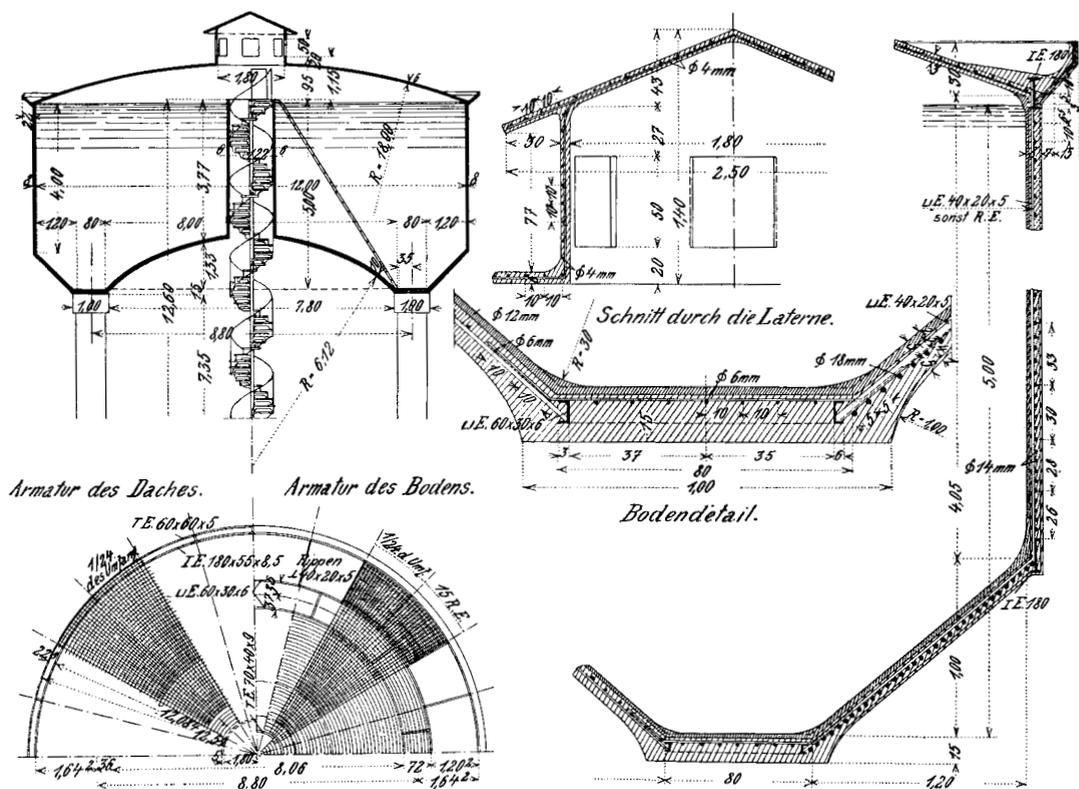
Theoretically, eq. 10-55 should always result in $t(z = 0) = 0$ when $z = 0$, but in practical terms a certain wall thickness t_0 with a steel cross-section $A_{s0} = t_0/n$ always results. For this reason, Wayss proposed a wall thickness t_0 with a steel-cross-section A_{s0} up to a height $z = z_0$, which, according to eq. 10-55, would produce the value t_0 , and only after that would the linear change in wall thickness down to the base of the tank be determined for $z = h$ according to eq. 10-55. For this latter section, Wayss specified a simple construction according to the intercept theorem (see Fig. 10-41):

$$\frac{t}{t_h} = \frac{z}{h} \quad (10-56)$$

Eq. 10-55 only takes into account the hoop tension stresses in the φ direction (see Fig. 10-39b); the normal stresses in the ϑ direction (see Fig. 10-39b) are not entered into the boiler formula.

The building of tanks etc. in reinforced concrete reached a new height after the 1890s. French building contractors became the leaders here, with about 10 companies competing to achieve the best form. In 1898 the company founded by Edmond Coignet (1856–1915) in 1890 set up two identical water tanks with a capacity of 500 m³ and a wall thickness of 8 cm (Fig. 10-42). This structure can be regarded as the prototype for reinforced concrete construction at the transition from the classical phase (1875–1900) to the accumulation phase (1900–1925) of theory of struc-

FIGURE 10-42
General arrangement and reinforcement drawings for the water tank at the Navy arsenal in Toulon [Wuczowski, 1910, p. 574]



tures; it formed, so to speak, the foundation for the genesis of reinforced concrete shells.

Coignet's monolithic water tank consists of several shells: a domed roof with central lantern light, internal and external cylindrical shells, a truncated cone shell and a domed base ("Intzeboden"). This reinforced concrete structure is supported on a masonry cylinder with 80 cm thick walls. The tank patented by Otto Intze (1843–1904) [Olbrisch, 1974] in 1883 is characterised by the fact that the horizontal thrust from the domed base is cancelled out by that from the truncated cone, meaning that the masonry cylinder is subjected to vertical forces only. This type of tank was widely used for storing water for railways, industry and waterworks. Coignet based the design of the two domes on membrane theory and specified rolled sections for their tension rings. For example, the upper tension ring was designed according to the equation

$$\sigma_{s, exist} = \frac{H \cdot r}{A_s} \leq \sigma_{s, permiss} = 1000 \text{ kg/cm}^2 \quad (10-57)$$

where A_s is the steel cross-section, r the radius of the dome on plan (6 m in this case) and H the following horizontal thrust due to membrane theory:

$$H = \frac{p \cdot (R - f)}{R} \quad (10-58)$$

with radius of curvature R (18 m in this case), rise of the dome f (0.95 m) and vertical support reaction p per metre due to the self-weight of the shell [Wuczkowski, 1910, pp. 575–576]. The derivation of eqs. 10-57 and 10-58 can also be found in the dissertation by Lutz Schöne [Schöne, 2011, pp. 48–49]. Coignet was unable to investigate how the membrane stress state is disturbed at the shell transitions, since engineering practice was ahead of theory formation and Coignet devised an elegant construction detail. Schöne carried out a structural analysis of Coignet's water tank in his dissertation and proved that the domes were adequately designed with respect to their static load-carrying capacity. He concludes that "the dome therefore exhibits high redundancy with respect to high loads, imperfect geometries or unintended situations during construction. This was certainly the reason why this type of tank could be built very economically" [Schöne, 2011, Annex 1, p. 9]. In Germany alone, more than 400 "Intze" water tanks were built between 1888 and 1904 [anon., 1905/1, p. 15] – most of them in steel. The economic "Intze" water tanks in reinforced concrete *à la* Coignet were now growing to be a serious rival to the steel tank, as the article by Richard Wuczkowski in the *Handbuch für Eisenbetonbau* (reinforced concrete manual) shows [Wuczkowski, 1910, pp. 574–578].

Reinforced concrete also started to be used for the building of gas tanks around the turn of the 20th century. Robert Maillart set a milestone with the two gas tanks built in St. Gallen, Switzerland, in 1902/1903. He was the first to consider the bending stresses due to M_θ , which he was able to obtain from an iterative graphical analysis (Fig. 10-43); the deflection curve of the tank wall in the meridional direction was determined with the help of Mohr's analogy. Taking the radius of curvature R of the deflection curve from the graphical analysis, Maillart calculated the bending moment

FIGURE 10-43
Graphical analysis of a gas tank
by Maillart [Wuczkowski, 1910, p. 485]

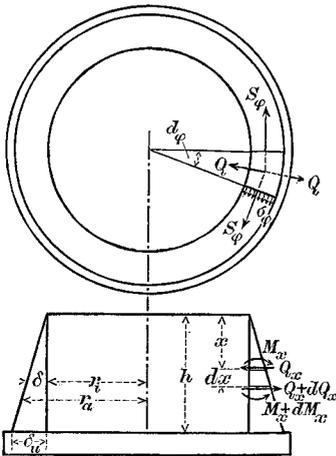
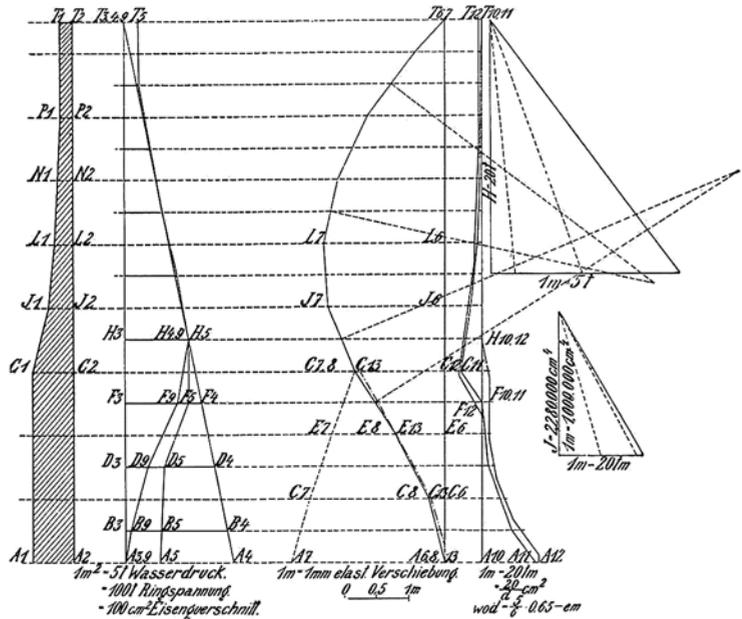


FIGURE 10-44
A cylindrical shell with a linearly
varying wall thickness and fixed at
the base [Reissner, 1908, p. 150]

Schwedler's comeback!

diagram using the familiar relationship $M_9 = (E \cdot I) / R$. Lutz Schöne endorsed Maillart's ingenious structural analysis in his dissertation – as Jörg Schlaich says – as “thinking in deformations” [Schöne, 2011, p. 51].

Shortly afterwards, Runge (1856–1927), a mathematician, published an approximation calculation for a cylindrical water tank with stepped wall thicknesses [Runge, 1904]. Picking up on this, Hans Reissner investigated a cylindrical shell with a linearly varying wall thickness (Fig. 10-44), which led to a fourth-order differential equation with varying coefficients with no closed-form solution. Reissner resolved this differential equation using power series and prepared it in the form of tables and charts.

But uncertainties still existed in the structural analysis of tanks etc. and this was expressed in the work of Emil Reich [Reich, 1907]. Reissner criticised not only Reich's awkward solution, but also his sample calculation, which results in a value for the wall thickness seven times the radius of the tank! Following Reissner's work, Federhofer proposed a graphical method for determining the stress distribution in cylindrical tank walls with any wall thickness [Federhofer, 1909, 1910]. The bending theory for cylindrical shells as a practical structural model for reinforced concrete tanks was fully developed by about 1915.

The first major step in the direction of a structural membrane theory for shells of revolution was taken by J. W. Schwedler in 1863 and 1866. He realised that in the structural analyses of domes it was not only the meridional stresses σ_θ that had to be quantified (as had been the case in the past), but also the hoop stresses σ_ϕ . Schwedler derived the equilibrium conditions for a dome-type shell of revolution with any geometry (see Fig. 10-39a) and specialised them for shallow shell surfaces and for spher-

rical surfaces [Schwedler, 1863]. In a further paper, he used his structural membrane theory for shells of revolution to calculate the member forces in the space frame he had invented – the Schwedler dome [Schwedler, 1863] (see section 9.1). As the internal forces in the radial and tangential directions of rotationally symmetric membrane shells can be determined from the equilibrium conditions alone, i.e. this is an internally statically determinate system, graphical analysis was already being used to analyse such loadbearing systems in the late 1870s (see section 7.3.5.1). In the *Monier-Broschüre*, Schwedler's membrane theory was used to design dome-type reinforced concrete shells [Wayss, 1887, pp. 31–33]. For the dome with radius of curvature R , the meridional stress per unit length of the circumference is

$$\sigma_{\vartheta} = p \cdot R \frac{1}{(1 + \cos \vartheta)} \quad (10-59)$$

and the hoop stress per unit length of the meridian is

$$\sigma_{\varphi} = p \cdot R \cdot \left[\cos \vartheta - \frac{1}{(1 + \cos \vartheta)} \right] \quad (10-60)$$

[Wayss, 1887, p. 32] (for designations see Fig. 10-39a). In both the above equations p is the weight per unit area of the dome surface including imposed loads, which is imposed in the radial direction equally throughout. Whereas the meridional stresses always lie within the compressive stress range, with a value of $0.5 \cdot p \cdot R$ at the crown and increasing towards the springings, the hoop stress changes its sign at $\vartheta = 51.83^\circ$, i.e. the hoop stresses are compressive at the top and tensile at the bottom. The tensile hoop stresses of a hemispherical dome have the value $\sigma_{\varphi} = -p \cdot R$ at the base, which is taken as the basis for the design. That results in the following steel reinforcement cross-sections [Wayss, 1887, S. 33]:

In the hoop direction per unit of length of the meridian

$$A_{s,\varphi} = \frac{p \cdot R}{\text{zul} \sigma_s} \quad (10-61)$$

and in the meridional direction per unit length of the circumference

$$A_{s,\vartheta} = \frac{t}{n} \quad (10-62)$$

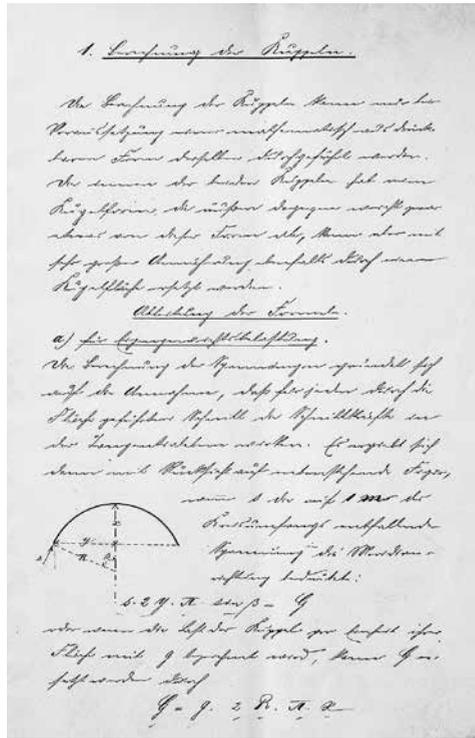
Eq. 10-62 (where t = shell thickness) is empirical because the denominator is given as $n > 1$. The reinforcement was laid in the radial and tangential directions. Reinforced concrete domes would be calculated according to this method up until the middle of the accumulation phase of theory of structures (1900–1925).

As part of his history of construction studies concerning the Bavarian Army Museum (1902–1904) and the Anatomical Institute (1905–1907) in Munich, Marco Pogacnik discovered the structural calculations for these buildings [Pogacnik, 2009]. Both were built by the Eisenbeton-Gesellschaft, a merger between Wayss & Freytag and Heilmann & Littmann which took place in 1903 with the aim of carrying out reinforced concrete projects in and around Munich. Fig. 10-45 shows the cover to the structural calculations for the dome at the Bavarian Army Museum, which were produced by Heilmann & Littmann.



FIGURE 10-45
(above left) Cover to the structural calculations dated 9 February 1903 for the dome at the Bavarian Army Museum, which were produced by Heilmann & Littmann [Pogacnik, 2009, p. 346]

FIGURE 10-46
(above right) Structural calculations by Emil Mörsch dated 15 April 1903 for the dome at the Bavarian Army Museum [Pogacnik, 2009, p. 348]



The inner and outer domes (16 m span) were to be built using the Hennebique system and consisted of ribs in the meridional and circumferential directions, i.e. consisted of curved T-beams. The following permissible stresses were assumed:

- steel in tension and compression: $\sigma_{s, \text{permiss}} = 1,000 \text{ kg/cm}^2$
- steel in shear: $\tau_{s, \text{permiss}} = 700 \text{ kg/cm}^2$
- concrete in compression: $\sigma_{b, \text{permiss}} = 25 \text{ kg/cm}^2$

An imposed load $p = 250 \text{ kg/m}^2$ and self-weight of the inner dome with decoration $g = 150 \text{ kg/m}^2$ was applied horizontally, resulting in a total load $q = 400 \text{ kg/m}^2$. However, this design was not built because, shortly before, Wayss & Freytag decided against the Hennebique system owing to the excessive licence fees (see section 10.2.2.3). Instead, Emil Mörsch from Wayss & Freytag submitted 22 pages of structural calculations for a totally new concept with two spherical reinforced concrete shells (Fig. 10-46).

Mörsch applied Schwedler’s membrane theory and assigned the forces to the T-section (40 and 45 mm deep) in the meridional and circumferential directions of the 6 cm thick shell. The shells of the Bavarian Army Museum can be interpreted as a further development of the Melan system (see section 10.2.2). Even the bolder, 22 m span, 10 cm thick dome with a rise $f = 5.75 \text{ m}$ is based on the Melan system. “The calculations were carried out according to the method for Schwedler domes for the various load cases during construction and in service” [Siegfried, 1908, p. 148]. Responsible for the calculations dated 17 May 1905 was not Mörsch this time, but Reiner from the Eisenbeton-Gesellschaft [Pogacnik, 2009, p. 352]. So

by about 1905, the calculation of reinforced concrete domes according to Schwedler's membrane theory had become established in the practical calculations of reinforced concrete engineers. Nevertheless, the analytical assessment of the transfer of the forces to the supports of the shell was still a closed book.

Heinrich Spangenberg (1879–1936), director of the Karlsruhe branch of Dyckerhoff & Widmann, working with Otto Mund, designed a pure membrane shell for the St. Blaise abbey church, the so-called St. Blaise Cathedral (Fig. 10-47). The overall structure has a diameter of 33.70 m, and the inner dome in the form of a membrane shell (1910–1913) spans 15.40 m with a rise $f = 1.49$ m and shell thickness $t = 8$ –12 cm [Spangenberg, 1912]. This shell, too, was calculated using the Schwedler method. The two engineers supported the shell on 20 radial struts integrated tangentially in the shell, essentially in keeping with the requirements of a membrane. The struts widen, haunch-like, around the edge of the shell so that the meridional stresses are grouped together as normal forces in the radial struts via the arching effect. By contrast, the hoop tensile stresses of the inner dome are carried by a continuous tension ring beam positioned around the edge of the shell. Here, too, the continuity principle for focusing the load

**Theory in practice:
the membrane shell of
St. Blaise Cathedral**

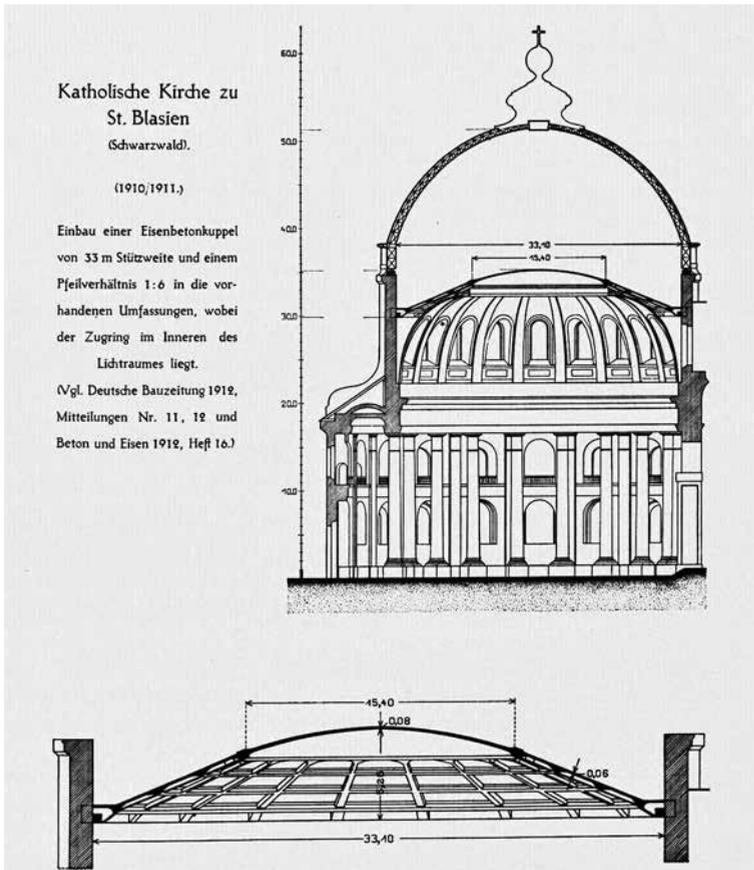


FIGURE 10-47
Inner dome of St. Blaise Cathedral
[Dyckerhoff & Widmann, 1920, p. 45]

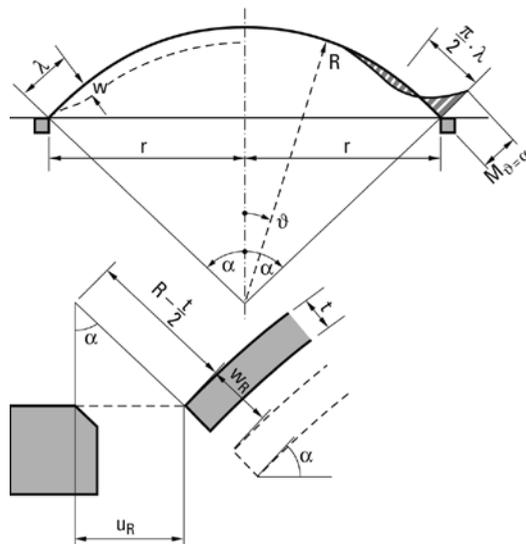
path was systematically implemented in the actual construction. The four 20-sided ring systems serve to brace the 20 radial struts and were analysed graphically together with the latter. There is a base ring at the springings with encased steel sections that have to carry a hoop tension force of 156 t. Therefore, the existing drum masonry on which the steel outer dome rests remains essentially unaffected by the horizontal forces of the radial struts to the inner dome.

The load-carrying system of the inner dome, which suits the membrane approach, required a complicated three-dimensional system of members. This system with its several degrees of static indeterminacy was analysed by Spangenberg and Mund with the help of the force method. In 1925 Franz Dischinger praised this shell design as “the boldest construction so far” [Dischinger, 1925, p. 362]. Nonetheless, it constituted an erratic element in reinforced concrete shells at that time. “The design was so closely based on a particular interpretation of membrane theory plus the specific conditions and restrictions that it cannot serve as a model for shells for buildings generally. So this approach, too, was only one step on the way to a better understanding of shells” [Schöne, 2011, p. 71].

Bending theory for shells of revolution takes shape

The fact that the membrane stress condition in shells is ‘disturbed’ by bending stresses at the supports was already well known by the middle of the accumulation phase of theory of structures (1900–1925). Fig. 10-48 illustrates this problem at the edge for the simplest case. Owing to the external loads (e.g. self-weight g), an elastic displacement of the dome w_R and a radial displacement of the base ring u_R ensues at the impost joint. As the impost joint may not open (compatibility condition), it must be closed by the meridional bending moment $M_{\vartheta=\alpha}$. The ensuing meridional bending moments M_{ϑ} decay like attenuated vibrations. It was the quantitative ascertainment of this disturbance at the edge in the form of the decay factor λ that finally led to a structural bending theory for shells.

FIGURE 10-48
Disturbed membrane stress state at the edge of a shell with constant thickness



ing theory [Strecker & Feldtkeller, 1929]. In his classic monograph on quadripole theory, Feldtkeller, who was appointed professor of electrical telecommunications technology at Stuttgart Technical University in 1936 following his work in Siemens' central laboratory in Berlin, systematically used the formal potential of matrix calculation for calculating linear electrical networks [Feldtkeller, 1937]. W. Quade finally provided an overview of the most important applications of matrix calculation for electrical networks and vibrations [Quade, 1940]. Feldtkeller's 1937 monograph helped the quadripole theory to become the showcase of matrix calculation in the fundamental engineering science disciplines. Two years later, the electrical engineer G. Kron, an employee of General Electric, published his book entitled *Tensor Analysis of Networks* [Kron, 1939]. Kron unfortunately mixed tensor and matrix theory. So the introduction of matrix calculation into electrical engineering experienced an unlucky start due to a number of less-than-fortunate publications [Zurmühl, 1950, p. 347]. Notwithstanding, Kron was able to cross the boundary between electrical engineering and mechanics. For example, he used the analogy between electrical and mechanical networks (elastic trusses) known to Maxwell and Kirchhoff for analysing three-dimensional trusses and formulated them in the language of matrix theory [Kron, 1944]. Kron's work inspired the aircraft engineer B. Langefors, an employee of the Swedish SAAB company, to summarise the force method in matrix form [Langefors, 1952]. Working independently, H. Falkenheiner published two articles in French [Falkenheiner, 1950, 1951], which Alf Samuelsson compared with the work of Langefors (1952): "The papers by Falkenheiner and Langefors are very similar. Both use the principle of deformation minimum according to Menabrea-Castigliano to deduce the matrix of influence coefficient expressing point displacements as a function of point loads. They also both describe a substructure technique. Langefors uses force in hypothetical cuts as redundants while Falkenheiner uses superposition coefficients of equilibrium systems as redundants. The method of Falkenheiner is then more general than that by Langefors" [Samuelsson, 2002, p. 7]. In 1953 Falkenheiner discussed his two articles in the light of the work of Langefors [Falkenheiner, 1953].

**The integration of matrix
formulation into engineering
mathematics**

11.5.3

One of the historical trails of matrix formulation in structural mechanics leads back to the Aerodynamics Department set up in 1925 by R. A. Frazer at the National Physics Laboratory in Teddington near London. Together with W. J. Duncan, Frazer researched the flutter of aircraft wings and in 1928 published the so-called *Flutter Bible* [Felippa, 2001]. Six years later, Duncan and A. R. Collar formulated conservative vibration problems in the language of matrix algebra [Duncan & Collar, 1934], and one year after that wrote a work on the motion equations of damped vibrations with the help of the powerful mathematical resources of matrix algebra [Duncan & Collar, 1935]. Looking back, Collar described this discovery of matrix algebra for a reformulation of vibration mechanics as follows: "Frazer had studied matrices as a branch of applied mathematics under Grace in Cambridge; and he recognized that the statement of, for example,

a ternary flutter problem in terms of matrices was neat and compendious. He was, however, more concerned with formal manipulation and transformation to other coordinates than with numerical results. On the other hand, Duncan and I were in search of numerical results for the vibration characteristics of airscrew blades; and we recognized that we could only advance by breaking the blade into, say, 10 segments and treating it as having 10 degrees of freedom. This approach also was more conveniently formulated in matrix terms, and readily expressed numerically. Then we found that if we put an approximate mode into one side of the equation, we calculated a better approximation on the other; and the matrix iteration procedure was born" [Collar, 1978, p.17]. The year 1938 saw Frazer, Duncan and Collar publish the first monograph in which areas of structural dynamics such as aeroelasticity were formulated systematically in

FIGURE 11-36
Eigenvalue analysis of a system of bars with three degrees of freedom after Frazer, Duncan and Collar [Frazer et al., 1963, p. 323]

Take as generalised coordinates q_1, q_2, q_3 the linear displacements of B, F, G , respectively. Then the displacement of D is $\frac{1}{2}(q_1 + q_2)$. In a general static displacement of the system, the elastic moments at A, C, E, D , will be $\frac{1}{3}q_1, \frac{2}{3}(q_1 + q_2), \frac{1}{3}q_2, \frac{1}{6}(q_3 - q_1 - q_2)$, and since the lever arms are all of unit length, the vertical forces at B, D, F, G are also $\frac{1}{3}q_1, \frac{2}{3}(q_1 + q_2), \frac{1}{3}q_2, \frac{1}{6}(q_3 - q_1 - q_2)$. To find the flexibility matrix, apply unit load at B, F, G in succession. When unit load is applied at B , we have by moments about AE ,

$$1 = \frac{1}{3}q_1 + \frac{2}{3}(q_1 + q_2) + \frac{1}{3}q_2 = q_1 + q_2,$$

while by moments about AB ,

$$\frac{2}{3}q_2 + \frac{2}{3}(q_1 + q_2) = 0, \text{ or } q_1 + 2q_2 = 0.$$

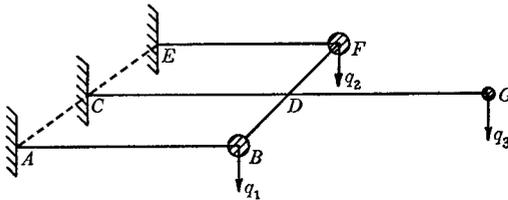


Fig. 10-8-1

Hence $q_1 = 2, q_2 = -1$, and since the moment at D is zero, $q_3 = q_1 + q_2 = 1$. The displacements are thus $\{2, -1, 1\}$. Similarly, when unit load is applied at F , the displacements are $\{-1, 2, 1\}$. When unit load is applied at G , we have by moments about AE ,

$$2 = \frac{1}{3}q_1 + \frac{2}{3}(q_1 + q_2) + \frac{1}{3}q_2 = q_1 + q_2,$$

and, since the displacement is symmetrical, $q_1 = q_2 = 1$. Moreover, by moments about BF ,

$$1 = \frac{1}{6}(q_3 - q_1 - q_2) \text{ or } q_3 = 11.$$

Hence in this case the displacements are $\{1, 1, 11\}$. The flexibility matrix is thus

$$\Phi = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & 1 & 11 \end{bmatrix}.$$

The inertia matrix is evidently

$$A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

terms of matrix algebra (Fig. 11-36) [Frazer et al., 1938]; since the end of the consolidation period of theory of structures (1900–1950), this has become a standard work for engineers who wish to find out something about solving vibration problems using matrices. Fig. 11-36 shows the eigenvalue analysis of a system of bars with the three degrees of freedom q_1 , q_2 and q_3 , which was investigated with the help of matrices; Fig. 11-36 is taken from the seventh unaltered reprint of the original edition of 1938. The monograph thus remained relevant until the middle of the innovation phase of theory of structures (1950–1975).

Zurmühl's monograph *Matrizen. Eine Darstellung für Ingenieure* (matrices – an explanation for engineers) of 1950 (Fig. 11-37) represented a milestone in the use of matrix formulation in the German-speaking countries. He realised that matrix formulation provided linear algebra with a means of expression that could be used to express the linear relationships prevailing in physics and the engineering sciences for operations that were uniform but difficult to present in customary mathematical language through equations of unsurpassed conciseness and clarity that always concentrate the user's attention on the essentials (see [Zurmühl, 1950, p. I]). Matrix theory will “assert itself more and more in engineering mathematics and perhaps soon play a similar role to vector theory, which today is indispensable” [Zurmühl, 1950, p. I]. Zurmühl's vision would very soon become reality as, during the 1950s, his monograph became the standard work on engineering mathematics. The book had been backed up since 1945 by the work of Alwin Walther (1898–1967), who tested numerical methods and procured obscure literature. It was at the Institute of Practical Mathematics (IPM), headed by Walther, at Darmstadt Technical University that Zurmühl investigated a matrix-based iteration method in the early 1940s, which he tested using the example of the calculations for a three-dimensional trussed framework with multiple degrees of static indeterminacy (see [Zurmühl, 1950, p. 282]).

Even before the Second World War, Walther's IPM was being called a “computations factory”, and in 1939 up to 70 female workers equipped with mechanical tabletop calculating machines were performing tasks associated with ballistics, lightweight construction, radiolocation and optics (see [Petzold, 1992, p. 226]). The thinking work of engineering science calculation had thus been schematised and divorced completely from the engineering work. What could have been more obvious than to automate this calculation work, as Zuse had suggested back in 1936?

Plans for a large, powerful, automatic program-controlled computing installation, which was to be assembled from parts for current calculating machines, were therefore discussed as early as 1943 at the IPM, which Walther had made available for research into wartime issues. Spurred on by the message concerning Aiken's large Mark I Automatic Sequence Controlled Calculator (ASSC), the generals of the German armed forces allocated the highest priority to Walther's project, which meant that he could procure the parts he needed to assemble the machine within a very short time. But a few days later the new installation disappeared into the bombed-out

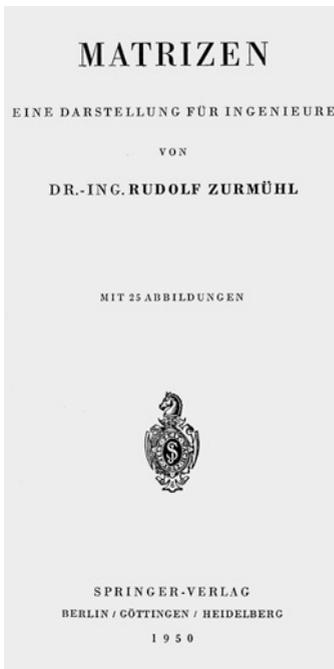


FIGURE 11-37
Title page of the first German book on the application of matrices to engineering and the engineering sciences

ruins of the IPM (see [Petzold, 1992, p. 228]). Through Prof. Herbert Wagner, manager of Special Department F at Henschel-Flugzeugwerke AG and, as such, Zuse's superior (Zuse had headed the structural analysis group since 1940), Walther first met Zuse in late 1942 [Zuse, 1993].

Wagner, that pioneer of aviation engineering and ingenious manipulator of numbers, had recognised the universal importance of Zuse's computer and had actively supported the project. Zuse wanted to work with Walther on his doctorate on the theme of the theory of general calculation. But Walther at that time regarded the computer primarily as a technical tool for rational engineering science calculations, in the sense of the numerical evaluation of formulas. Zuse's doctorate unfortunately remained only an outline. Petzold suspects that it would have proved difficult to carry out such work with Walther, who gave priority to analogue technology (see [Petzold, 1992, p. 197]).

11.5.4 A structural analysis matrix method: the carry-over method

Nevertheless, Walther, by promoting Zurmühl, had recognised the heuristic power of matrix formulation for physics and the fundamental engineering science disciplines. And therefore the Darmstadt doctorate project of H. Fuhrke on the determination of beam oscillations with the help of matrices could be completed in the early 1950s [Fuhrke, 1955].

Even more important for structural analysis was the carry-over method for calculating continuous beams with any number of spans created by S. Falk in 1956 [Falk, 1956], which translated the solution to the differential beam equation fully into the language of matrix formulation (Fig. 11-38). The carry-over method only exists through matrix operations and in the case of continuous beams leads to systems with a maximum of two linear equations. The degree of static or geometric indeterminacy does not appear in the carry-over method, which belongs to the group of reduction methods; far more significant are the topological properties of the structural system. Consequently, the dual nature of theory of structures – due to the force and displacement methods – is insignificant in the carry-over method.

Joachim Scheer was probably the first engineer in the German-speaking countries to investigate in detail the use of program-controlled automatic calculators for structural tasks in conjunction with the carry-over method [Scheer, 1958]. The program presented by Scheer in 1958 was employed for practical tasks, e. g. a number of projects for the engineering practice of Dr. Homberg in Hagen [Scheer, 1998]. Scheer told the author in 1998 that his dissertation on the problem of the overall stability of singly-symmetric I-beams published in the journal *Der Stahlbau* in 1959 had only been rendered possible through the use of the carry-over method and computers in 1957/1958 [Scheer, 1998]. Despite this, the influence of the carry-over method, like other reduction methods, remained limited in the theory and practice of structural analysis because matrix analysis covered only some of the structural systems. At the same time, Klöppel and Scheer employed matrix analysis successfully for preparing the programming of the buckling theory of stiffened rectangular steel plates ac-

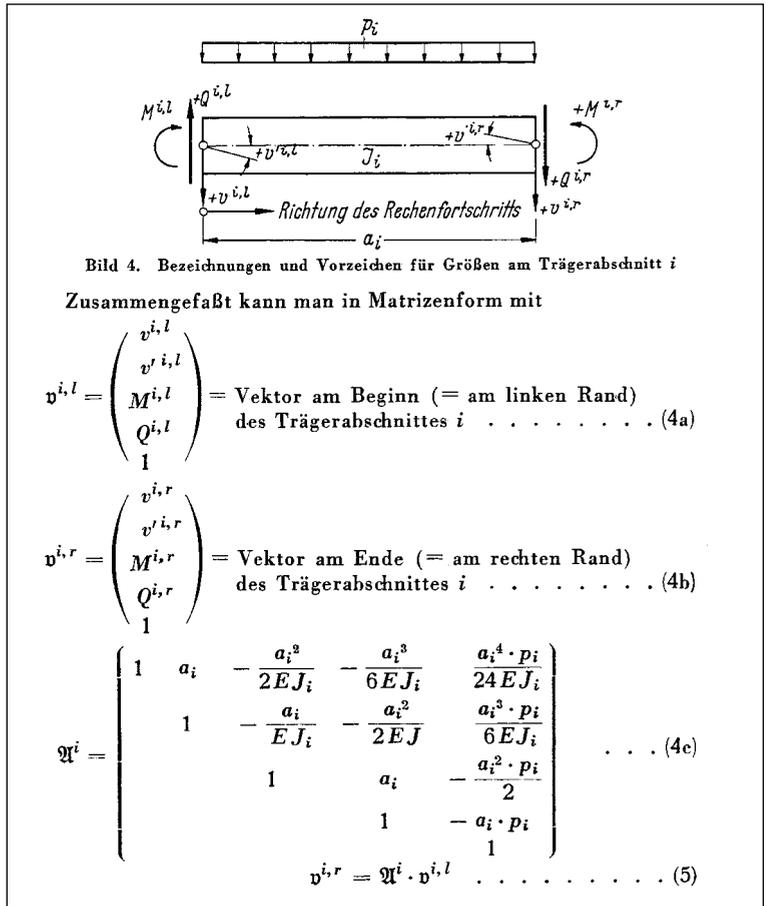


FIGURE 11-38
 Carry-over method after Falk
 in the representation by Scheer
 [Scheer, 1958, p. 228]

According to the energy method. With the help of the IBM 704 computer donated to Darmstadt Technical University by IBM Deutschland in 1958, it was possible to calculate the buckling values of standard stiffened rectangular plate cases from the buckling matrix in a relatively short time and publish these as design charts for everyday structural steelwork calculations [Klöppel & Scheer, 1960]; a second volume followed eight years later [Klöppel & Möller, 1968]. Such design charts for the stress analyses of plate and shell structures calculated with the help of sophisticated research programmes provided important assistance in the production of structural calculations carried out partly by hand and partly with the computer even after the innovation phase of theory of structures (1950–1975).

The carry-over method was the historico-logical starting point of structural matrix analysis. This fact is revealed by numerous publications that appeared in the middle of the innovation phase of theory of structures (1950–1975), one example of which was *Matrix Methods in Elastomechanics* (Fig. 11-39). The cover shows a transformation matrix for rotating the system of coordinates through an angle γ about the z axis.

The carry-over method was suitable for manual and computerised calculations; this latter point had already been mentioned by S. Falk in 1956

(see [Falk, 1956, p. 231]). The carry-over method could be used to multiply an $m \times r$ matrix (left) by an $r \times n$ matrix (right) in a particularly simple and clear fashion according to the scheme introduced by Falk [Falk, 1951]. The $r \times n$ matrix is positioned to the right above the $m \times r$ matrix such that the extended n columns of the $r \times n$ matrix and the extended m rows of the $m \times r$ matrix overlap to form the result matrix, the $m \times n$ matrix. For example, the element in the i th row and k th column of the result matrix is calculated from the sum of the products of the respective elements in the i th row of the $m \times r$ matrix and the associated elements in the k th column of the $r \times n$ matrix. Fig. 11-40 shows a numerical example of a matrix multiplication according to the Falk scheme. The $m \times r$ matrix ($m = 3, r = 2$) is to be multiplied from the right by the $r \times n$ matrix ($r = 2, n = 4$). The element in the third line and third column of the $m \times n$ result matrix ($m = 3, n = 4$) then becomes $(6 \times 6) + (1 \times 8) = 36 + 8 = 44$. In the Falk scheme the arithmetisation of the matrix calculation for the purpose of programming is obvious; the suitability of the Falk scheme for manual calculations does not contradict this, but ensures that manual calculations, too, undergo further formalisation. Therefore, the prescriptive use of symbols became ever more established in the everyday work of the practising structural engineer.

The carry-over method is a method for solving linear differential equations of the order $2n$ ($n = 1, 2, 3, 4, \dots$). The only difference is that the carry-over method is formulated in the language of matrix algebra. Christian Petersen extended the carry-over method significantly. Examples of his work are his derivatives of the transformation matrices for the beam on continuous elastic supports [Petersen, 1965], the curved beam [Petersen, 1966/2] and the circular curved beam on elastic supports [Petersen, 1967]. Nevertheless, the carry-over method is not suitable for solutions with a severely decaying character such as the beam on elastic supports. On the other hand, the carry-over method supplies reliable results when investigating beams with a high bending stiffness. For example, Petersen was the first to specify the right transformation matrices for calculating the eigenfrequencies and eigenmodes of guyed masts modelled as continuous beams on elastic supports [Petersen, 1970]. He established that the shear force Q and the normal force N belonging to the orthogonal section were taken instead of the transverse force T_{iR} and the longitudinal force D_i (from the transverse section), which is totally wrong when formulating the boundary and transfer conditions at the elastic spring supports. Therefore, in his later study on the themes of second-order theory, and also for overturning, torsional-flexural buckling and buckling problems, Petersen derived the basic equations and their solutions always using transverse sections (Fig. 11-41).

In his habilitation thesis on the vibrations of tower-like structures taking particular account of an attenuation model independent of frequency and stochastic excitation [Petersen, 1971], Petersen determined transformation matrices for a series of problems. This thesis concerns the development of a carry-over method for calculating externally excited

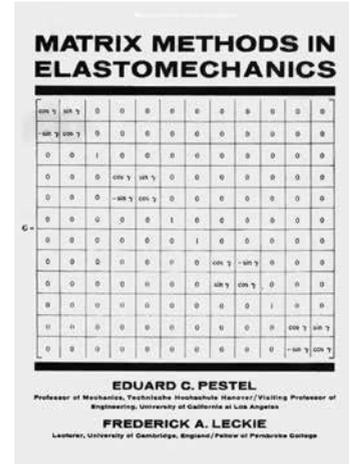


FIGURE 11-39 Cover of the pocket-book edition of *Matrix Methods in Elastomechanics* [Pestel & Leckie, 1963]

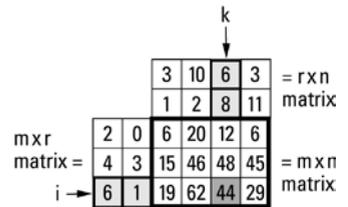
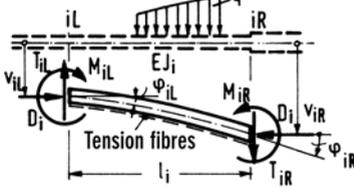


FIGURE 11-40 Numerical example of matrix multiplication according to the Falk scheme

Transformation matrices method, second-order theory

Definition of deformations and stress resultants



Bar factor: $\epsilon = l \sqrt{\frac{D}{EJ}}$ D (compression)

Transformation matrix:

$$[F] = \begin{bmatrix} f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & f_{22} & f_{23} & f_{24} \\ f_{31} & f_{32} & f_{33} & f_{34} \\ f_{41} & f_{42} & f_{43} & f_{44} \end{bmatrix} \quad [F]^q = \begin{bmatrix} f_{1q} \\ f_{2q} \\ f_{3q} \\ f_{4q} \end{bmatrix}$$

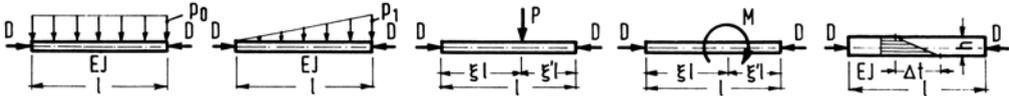
Elements of field matrix F ($EJ = \text{const.}$), first-order theory:

$$[F] = \begin{bmatrix} 1 & l & -\frac{1}{2} \cdot \frac{l^2}{EJ} & -\frac{1}{6} \cdot \frac{l^3}{EJ} \\ 0 & 1 & -\frac{l}{EJ} & -\frac{1}{2} \cdot \frac{l^2}{EJ} \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Second-order theory:

$$[F] = \begin{bmatrix} 1 & \frac{\sin \epsilon}{\epsilon} \cdot l & -\frac{1 - \cos \epsilon}{\epsilon^2} \cdot \frac{l^2}{EJ} & -\frac{\epsilon - \sin \epsilon}{\epsilon^3} \cdot \frac{l^3}{EJ} \\ 0 & \cos \epsilon & -\frac{\sin \epsilon}{\epsilon} \cdot \frac{l}{EJ} & -\frac{1 - \cos \epsilon}{\epsilon^2} \cdot \frac{l^2}{EJ} \\ 0 & \epsilon \sin \epsilon \cdot \frac{EJ}{l} & \cos \epsilon & \frac{\sin \epsilon}{\epsilon} \cdot l \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Elements of load vectors F^q :



First-order theory:

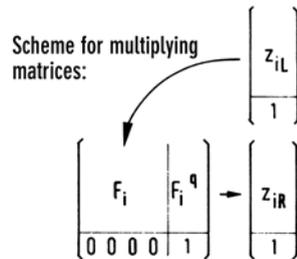
$$p_0 \cdot \begin{bmatrix} \frac{1}{24} \cdot \frac{l^4}{EJ} \\ \frac{1}{6} \cdot \frac{l^3}{EJ} \\ -\frac{1}{2} \cdot l^2 \\ -1 \end{bmatrix} \quad p_1 \cdot \begin{bmatrix} \frac{1}{120} \cdot \frac{l^4}{EJ} \\ \frac{1}{24} \cdot \frac{l^3}{EJ} \\ -\frac{1}{\epsilon} \cdot l^2 \\ -\frac{1}{2} \cdot l \end{bmatrix} \quad P \cdot \begin{bmatrix} \frac{\xi^3}{6} \cdot \frac{l^3}{EJ} \\ \frac{\xi^2}{2} \cdot \frac{l^2}{EJ} \\ -\xi \cdot l \\ -1 \end{bmatrix} \quad M \cdot \begin{bmatrix} -\frac{\xi^2}{2} \cdot \frac{l^2}{EJ} \\ -\xi \cdot \frac{l}{EJ} \\ 1 \\ 0 \end{bmatrix} \quad \alpha_t \frac{\Delta t}{h} \cdot \begin{bmatrix} -\frac{l^2}{2} \\ -l \\ 0 \\ 0 \end{bmatrix}$$

Second-order theory:

$$p_0 \cdot \begin{bmatrix} \frac{\epsilon^2 - 2(1 - \cos \epsilon)}{2\epsilon^4} \cdot \frac{l^4}{EJ} \\ \frac{\epsilon - \sin \epsilon}{\epsilon^3} \cdot \frac{l^3}{EJ} \\ -\frac{1 - \cos \epsilon}{\epsilon^2} \cdot l^2 \\ -1 \end{bmatrix} \quad p_1 \cdot \begin{bmatrix} \frac{\epsilon^3 - 6(\epsilon - \sin \epsilon)}{6\epsilon^5} \cdot \frac{l^4}{EJ} \\ \frac{\epsilon^2 - 2(1 - \cos \epsilon)}{2\epsilon^4} \cdot \frac{l^3}{EJ} \\ -\frac{\epsilon - \sin \epsilon}{\epsilon^3} \cdot l^2 \\ -\frac{l}{2} \end{bmatrix} \quad P \cdot \begin{bmatrix} \frac{\epsilon \xi^3 - \sin \epsilon \xi^3}{\epsilon^3} \cdot \frac{l^3}{EJ} \\ \frac{1 - \cos \epsilon \xi'}{\epsilon^2} \cdot \frac{l^2}{EJ} \\ -\frac{\sin \epsilon \xi'}{\epsilon} \cdot l \\ -1 \end{bmatrix} \quad M \cdot \begin{bmatrix} -\frac{1 - \cos \epsilon \xi'}{\epsilon^2} \cdot \frac{l^2}{EJ} \\ -\frac{\sin \epsilon \xi'}{\epsilon} \cdot \frac{l}{EJ} \\ \cos \epsilon \xi' \\ 0 \end{bmatrix} \quad \alpha_t \frac{\Delta t}{h} \cdot \begin{bmatrix} -\frac{(1 - \cos \epsilon)}{\epsilon^2} \cdot l^2 \\ -\frac{\sin \epsilon}{\epsilon} \cdot l \\ -EJ(1 - \cos \epsilon) \\ 0 \end{bmatrix}$$

Scheme for transformation from interface iL to interface iR : $[z]_{iR} = [F]_i [z]_{iL} + [F]_i^q$. $[z]_i$ is the state vector that includes the deformations and stress resultants at interface i :

$$[z]_i = \begin{bmatrix} v_i \\ \varphi_i \\ M_i \\ T_i \end{bmatrix} \begin{array}{l} \text{Deflection} \\ \text{Angle of rotation due to bending} \\ \text{Bending moment} \\ \text{Transverse force} \end{array}$$



attenuated beam vibrations according to second-order theory, for which he specifies the complex transformation matrix [Petersen, 1971, pp. 95–100]. “In the meaning of mathematics,” Petersen writes, “the carry-over method achieves exact solutions for various individual problems – something that no FEM calculations achieve. My intention at that time, to write a book about the method of transformation matrices, was abandoned again as the ‘heavy-calibre’ FEM started to assert itself” [Petersen, 2017, p. 3]. The “heavy-calibre FEM” would first become practically effective as computational statics within the scope of computational mechanics during the diffusion phase of theory of structures (1975 to date).

FIGURE 11-41 (PAGE 844)
Transformation matrices for trusses according to first- and second-order theory on the basis of transverse internal forces T_{iR} and D_i [Petersen, 1980, p. 202]

BESTELLSCHEIN

| Stück | Bestell-Nr.: | Titel | Preis* € |
|---|-------------------|---|-----------|
| | 978-3-433-03229-9 | The History of the Theory of Structures | 149,- |
| | 909857 | Gesamtverzeichnis Ernst & Sohn 2017/2018 | kostenlos |
| Monatlicher E-Mail-Newsletter: Anmeldung unter www.ernst-und-sohn.de/newsletter | | | |

Liefer- und Rechnungsanschrift: privat geschäftlich

| | | | |
|-------------------------|---|-----|---------|
| Firma | | | |
| Ansprechpartner | | | Telefon |
| UST-ID Nr. / VAT-ID No. | | | Fax |
| Straße//Nr. | | | E-Mail |
| Land | - | PLZ | Ort |

Vertrauensgarantie: Dieser Auftrag kann innerhalb von zwei Wochen beim Verlag Ernst & Sohn, Wiley-VCH, Boschstr. 12, D-69469 Weinheim, schriftlich widerrufen werden.

Datum / Unterschrift

Wilhelm Ernst & Sohn
 Verlag für Architektur und
 technische Wissenschaften
 GmbH & Co. KG
 Rotherstraße 21, 10245 Berlin
 Deutschland
www.ernst-und-sohn.de

*€-Preise gelten ausschließlich in
 Deutschland. Alle Preise enthalten die
 gesetzliche Mehrwertsteuer. Die Lieferung
 erfolgt zuzüglich Versandkosten. Es gelten
 die Lieferungs- und Zahlungsbedingungen
 des Verlages. Irrtum und Änderungen
 vorbehalten.

Stand: Dezember 2017
 (homepage_Probekapitel)

