

Hugo Hens

Building Physics – Heat, Air and Moisture

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ABOUT THE BOOK

Bad experiences with construction quality the energy crises of 1973 and 1979, complaints about „sick buildings“, thermal acoustical, visual and olfactory discomfort the need for good air quality the move towards energy efficiency decarbonization and sustainability – all these have accelerated the development of a discipline that for a long time was hardly more than an academic exercise: building physics. The discipline embraces domains such as heat and mass transfer building acoustics lighting, indoor environmental quality energy efficiency and, in some countries fire safety. Through the application of physical knowledge and its combination with information coming from other disciplines building physics helps to understand the physical phenomena governing building parts building envelope whole building and built environment performance – called urban physics. Today building physics has become a key player on the road to highly performing new buildings and renovations.

This book deals with heat air and moisture transport in building parts or assemblies and whole buildings with emphasis on the building engineering applications. Compared to the third edition this fourth edition has been expanded in chapter 1 to include the physical determination of the thermal conductivity of materials together with an in-depth discussion of all the effects of thicker insulation layers. In chapter 2, additional information has been added on wind pressure and the evaluation of condensation inside the building components while a new chapter 4 on material properties has been included. The whole book including the figures has been revised and restructured where necessary.

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Contents

Preface	<i>xv</i>
About the Author	<i>xix</i>
List of Units and Symbols	<i>xxi</i>

0	Introduction	<i>1</i>
0.1	Subject of the Book	<i>1</i>
0.2	Building Physics?	<i>1</i>
0.2.1	Definition	<i>1</i>
0.2.2	Constraints	<i>2</i>
0.2.2.1	Comfort	<i>2</i>
0.2.2.2	Health and Well-being	<i>3</i>
0.2.2.3	Architecture and Materials	<i>3</i>
0.2.2.4	Economy	<i>3</i>
0.2.2.5	Sustainability	<i>3</i>
0.3	Importance?	<i>4</i>
0.4	History	<i>5</i>
0.4.1	In General	<i>5</i>
0.4.2	Applied Physics	<i>5</i>
0.4.2.1	Heat, Air, Moisture	<i>5</i>
0.4.2.2	Acoustics	<i>8</i>
0.4.2.3	Lighting	<i>9</i>
0.4.3	Indoor Air Quality and Thermal Comfort	<i>9</i>
0.4.4	Building Services	<i>11</i>
0.4.5	Building Design and Construction	<i>11</i>
0.4.6	Hall of Fame	<i>12</i>
0.4.7	Building Physics at the KULeuven and Other Universities in the Low Countries	<i>13</i>
	Further Reading	<i>15</i>
1	Heat Transfer	<i>17</i>
1.1	In General	<i>17</i>
1.1.1	Heat	<i>17</i>
1.1.1.1	What?	<i>17</i>
1.1.1.2	Sensible Heat	<i>17</i>

1.1.1.3	Latent Heat	18
1.1.2	Temperature	18
1.1.3	Why are Heat and Temperature so Compelling?	18
1.1.4	Some Definitions	19
1.2	Conduction	19
1.2.1	Conservation of Energy	19
1.2.2	Conduction Laws	20
1.2.2.1	First Law	20
1.2.2.2	Second Law	22
1.2.3	Thermal Conductivity	22
1.2.3.1	In General	22
1.2.3.2	Heat Transfer Modes Fixing the Property	22
1.2.4	Steady-State	26
1.2.4.1	What?	26
1.2.4.2	One Dimension, Flat Assemblies	27
1.2.4.3	Two Dimensions, Cylinder Symmetric	33
1.2.4.4	Two and Three Dimensions: Thermal Bridges	35
1.2.5	Non-steady-state	38
1.2.5.1	In General	38
1.2.5.2	Periodic Boundary Conditions, Flat Assemblies	39
1.2.5.3	Any Boundary Conditions, Flat Assemblies	48
1.2.5.4	Two and Three Dimensions: Thermal Bridges	52
1.3	Heat Exchange at Surfaces by Convection and Radiation	52
1.3.1	What?	52
1.3.2	Convection	53
1.3.2.1	In General	53
1.3.2.2	Typology	55
1.3.2.3	Quantifying the Convective Surface Film Coefficient	55
1.3.2.4	Values for the Convective Surface Film Coefficient	58
1.3.3	Radiation	63
1.3.3.1	In General	63
1.3.3.2	Definitions	63
1.3.3.3	Reflection, Absorption and Transmission	64
1.3.3.4	Radiant Surfaces	66
1.3.3.5	Simple Formulae	74
1.4	Building-related Applications	76
1.4.1	Surface Film Coefficients and Reference Temperatures	76
1.4.1.1	Methodology	76
1.4.1.2	Indoors	76
1.4.1.3	Outdoors	78
1.4.2	Steady-state, Flat Assemblies	80
1.4.2.1	Thermal Transmittance of Envelope Assemblies, Partitions and Party Walls	80
1.4.2.2	Average Thermal Transmittance of Envelope Parts in Parallel	83
1.4.2.3	Electrical Analogy	84

- 1.4.2.4 Thermal Resistance of Non-ventilated Cavities 84
- 1.4.2.5 Interface Temperatures 86
- 1.4.2.6 Effect of Ever Thicker Insulation Layers on the Thermal Transmittance 87
- 1.4.2.7 Solar Transmittance 88
- 1.4.3 Local Inside Surface Film Coefficients 91
- 1.4.4 Steady-state: Two and Three Dimensions 92
 - 1.4.4.1 Pipes 92
 - 1.4.4.2 Floors on Grade 93
 - 1.4.4.3 Thermal Bridges 94
 - 1.4.4.4 Windows 98
 - 1.4.4.5 Building Envelopes 99
- 1.4.5 Heat Balances 100
- 1.4.6 Non-steady-state 101
 - 1.4.6.1 Periodic Boundary Conditions: Flat Assemblies 101
 - 1.4.6.2 Periodic Boundary Conditions: Spaces 101
 - 1.4.6.3 Any Boundary Conditions: Thermal Bridges 105
 - Problems and Solutions 106
 - Further Reading 118
- 2 Mass Transfer 121**
 - 2.1 In General 121
 - 2.1.1 Facts 121
 - 2.1.2 Definitions 122
 - 2.1.3 Saturation Degree Scale 123
 - 2.1.4 Air and Moisture Transfer 123
 - 2.1.5 Moisture Sources 125
 - 2.1.6 Air and Moisture in Relation to Durability 127
 - 2.1.7 Links with Energy Transfer 127
 - 2.1.8 Conservation of Mass 128
 - 2.2 Air 129
 - 2.2.1 In General 129
 - 2.2.2 Air Pressure Differentials 130
 - 2.2.2.1 Wind 130
 - 2.2.2.2 Stack Effect 130
 - 2.2.2.3 Fans 133
 - 2.2.3 Air Permeability and Air Permeances 133
 - 2.2.4 Airflow in Open-porous Materials 135
 - 2.2.4.1 The Conservation Law Adapted 135
 - 2.2.4.2 One Dimension: Flat Assemblies 138
 - 2.2.4.3 Two and Three Dimensions 139
 - 2.2.5 Airflow Through Assemblies with Air-open Layers, Leaky Joints, Leaks, Cavities, etc. 140
 - 2.2.6 Airflow at the Building Level 141
 - 2.2.6.1 Definitions 141

- 2.2.6.2 Thermal Stack 142
- 2.2.6.3 Large Openings 142
- 2.2.6.4 The Conservation Law Applied 143
- 2.2.6.5 Applications 145
- 2.2.7 Combined Heat and Airflow Through Assemblies Composed of Open-porous Layers 148
 - 2.2.7.1 Heat Balance 148
 - 2.2.7.2 Steady-state: Flat Assemblies 148
 - 2.2.7.3 Steady-state, Two and Three Dimensions 152
 - 2.2.7.4 Non-steady-state, Flat Assemblies 152
 - 2.2.7.5 Non-steady-state, Two and Three Dimensions 153
 - 2.2.7.6 Air-permeable Layers, Joints and Leaks 153
 - 2.2.7.7 Vented Cavities 153
- 2.3 Water Vapour 156
 - 2.3.1 Water Vapour in the Air 156
 - 2.3.1.1 In General 156
 - 2.3.1.2 Quantities 156
 - 2.3.1.3 Vapour Saturation Pressure 157
 - 2.3.1.4 Relative Humidity 157
 - 2.3.1.5 Changes of State in Humid Air 161
 - 2.3.1.6 Enthalpy of Humid Air 162
 - 2.3.1.7 Measuring Air Humidity 162
 - 2.3.2 Vapour Balance in Spaces 163
 - 2.3.3 Relative Humidity On Inside Surfaces 165
 - 2.3.4 Vapour in Open-porous Materials 168
 - 2.3.4.1 Different from Air? 168
 - 2.3.4.2 Sorption/Desorption Isotherm 168
 - 2.3.5 Vapour Transfer in the Air 172
 - 2.3.6 Vapour Flow by Diffusion in Open-porous Materials and Building Assemblies 174
 - 2.3.6.1 Flow Equation 174
 - 2.3.6.2 Vapour Resistance Factor μ 175
 - 2.3.6.3 Mass Conservation 176
 - 2.3.6.4 Applicability of the <Equivalent> Diffusion Concept 177
 - 2.3.6.5 Steady State: Flat Assemblies 177
 - 2.3.6.6 Steady State: Two and Three Dimensions 186
 - 2.3.6.7 Non-steady State 187
 - 2.3.7 Vapour Flow by Diffusion and Moist Air Moving Through Open-porous Assemblies 189
 - 2.3.7.1 In General 189
 - 2.3.7.2 Isothermal, Single- and Multi-layered Assemblies 190
 - 2.3.7.3 Non-isothermal, Single- and Multi-layered Assemblies 191
 - 2.3.8 Surface Film Coefficients for Diffusion 195
 - 2.3.8.1 Derivation 195
 - 2.3.8.2 Applications 198

2.3.9	Evaluating Interstitial Condensation in Practice	201
2.3.9.1	Boundary Conditions Used	201
2.3.9.2	Calculation Sequence	203
2.3.9.3	Example	204
2.4	Moisture	209
2.4.1	In General	209
2.4.2	Water Flow in a Pore	209
2.4.2.1	Capillarity	209
2.4.2.2	Poiseuille's Law	211
2.4.2.3	Isothermal Water Flow in a Pore Contacting Water	212
2.4.2.4	Isothermal Water Flow in a Pore After Water Contact	218
2.4.2.5	Non-isothermal Water Transfer in a Pore After Water Contact	218
2.4.2.6	Remark	219
2.4.3	Vapour Flow in a Pore Containing Water Isles with Air Inclusions in Between	219
2.4.3.1	A Short Description	219
2.4.3.2	Isothermal	219
2.4.3.3	Non-isothermal	220
2.4.4	Moisture Flow in and Through Materials and Assemblies	221
2.4.4.1	Transport Equations	221
2.4.4.2	Moisture Permeability	223
2.4.4.3	Mass Conservation	223
2.4.4.4	Starting, Boundary and Contact Conditions	224
2.4.4.5	Remarks	224
2.4.5	Simple Moisture Flow Model	225
2.4.5.1	How to Do?	225
2.4.5.2	Applying the Simple Model	227
	Problems and Solutions	240
	Further Reading	261
3	Heat, Air and Moisture Combined	265
3.1	Why?	265
3.2	Material and Assembly Level	265
3.2.1	Assumptions	265
3.2.2	Solution	266
3.2.3	Conservation of Mass	266
3.2.4	Conservation of Energy	267
3.2.5	Flux Equations	270
3.2.5.1	Heat	270
3.2.5.2	Mass, Air	270
3.2.5.3	Mass, Moisture	270
3.2.5.4	Remark	271
3.2.6	Equations of State	271
3.2.6.1	Enthalpy and Vapour Saturation Pressure in Relation to Temperature	271

3.2.6.2	Relative Humidity in Relation to Moisture Content	271
3.2.6.3	Suction in Relation to Moisture Content	271
3.2.7	Start, Boundary and Contact Conditions	271
3.2.8	Two Examples of Simplified Models	272
3.2.8.1	Assemblies Composed of Non-Hygroscopic, Non-Capillary Materials	272
3.2.8.2	Assemblies Composed of Fine Porous, Hygroscopic Materials	274
3.3	Whole Building Level	274
3.3.1	In General	274
3.3.2	Balance Equations	275
3.3.2.1	Vapour	275
3.3.2.2	Air	276
3.3.2.3	Heat	276
3.3.2.4	Closing the Loop	278
3.3.3	Sorption Active Surfaces and Hygric Inertia	279
3.3.3.1	In General	279
3.3.3.2	Sorption-Active Thickness	280
3.3.3.3	Zone with One Sorption-Active Surface	282
3.3.3.4	Zone with Several Sorption-Active Surfaces	283
3.3.3.5	Harmonic Analysis	284
3.3.4	Consequences	284
	Problems and Solutions	287
	Further Reading	300
4	Heat, Air and Moisture Material Property Values	303
4.1	In General	303
4.2	Dry Air and Water	304
4.3	Thermal Properties	305
4.3.1	Definitions	305
4.3.2	Standard Values	305
4.3.2.1	Regardless of Being on the In- or on the Outside of the Thermal Insulation	305
4.3.2.2	Depending on Being on the In- or on the Outside of the Thermal Insulation	309
4.3.3	Surfaces, Radiant Properties	316
4.3.4	Measured Values	317
4.3.4.1	Thermal Conductivity, Test Methods	317
4.3.4.2	Test Results	318
4.4	Air Properties	325
4.4.1	Standard Values	325
4.4.2	Measured Values	325
4.4.2.1	Air Permeance, Test Method	325
4.4.2.2	Test Results	327
4.5	Moisture Properties	336
4.5.1	Standard Values	336

4.5.1.1	Building and Finishing Materials	336
4.5.1.2	Insulation Materials	340
4.5.2	Measured Values	340
4.5.2.1	Diffusion Resistance Factor (μ), Test Method	340
4.5.2.2	Test Results	342
	Further Reading	357

Postscript 359

Index 361

Preface

Until the energy crisis of 1973, building physics was a dormant beauty within building engineering, with seemingly limited applicability in practice. While soil mechanics, structural mechanics, construction materials, building itself and heating, ventilation, air conditioning (HVAC) were perceived as essential, designers only demanded advice on room acoustics, moisture tolerance, summer overheating or lighting when really needed or when in newly occupied buildings problems arose. Energy was no concern, while thermal comfort and indoor environmental quality were presumably guaranteed thanks to air infiltration, window opening and the HVAC system. 1973 and the energy crisis of 1979, persisting moisture problems, complaints about sick buildings, thermal, visual and olfactory discomfort, the move to more sustainability and, since the 1980s, global warming with today the quest for carbon neutrality changed this all. Besides, the pressure to diminish energy use and carbon emitted without degrading building usability more than activated the importance of a performance-based building and building part design and construction. As a result, building physics and related potentiality to quantify performances moved to the frontline of building innovation.

Like all engineering sciences, building physics is oriented towards application. This demands a sound knowledge of the basics in each of its branches: heat and mass transfer, acoustics, lighting, energy and indoor environmental quality. Advancing the basics on heat and mass transfer is the main objective of this volume, be it for mass flow limited to air, water vapour and moisture. In the introduction, building physics as a discipline is sketched and its history is given. The first chapter then concentrates on heat transport, with conduction, convection and radiation as main topics, followed by common concepts linked to and applications in the field of building and building part or assembly design and construction. The second chapter treats mass transport, with air, water vapour and moisture as the main topics. Also here, attention goes to the concepts and applications related to whole buildings and building parts. The third chapter discusses combined heat, air and moisture transport. All chapters end with exercises. In the fourth chapter, standard lists with heat, air and moisture material properties and measured data are given.

This content is the result of 38 years of teaching building physics to architectural, building and civil engineering students, that, coupled to more than 36 years of experience in building and building part performance research and more than 50 years

of activity in consultancy and in curing hundreds of heat, air and moisture-related damage cases. When and where needed, information from international sources and literature has been consulted, which is why all chapters end with an extended list of references and further reading. The book uses SI units. It could be of help for undergraduate and graduate students in architectural and building engineering, although also students in mechanical engineering studying HVAC and practising building engineers, who want to refresh their knowledge, may benefit. Presumed is the reader has a sound knowledge of calculus and differential equations along with a background in physics, thermodynamics, hydraulics, construction materials and building design and construction.

Compared to the third edition published in 2017, the book has been reorganised, corrected, revised and expanded where appropriate for this fourth edition.

Acknowledgements

The book reflects the work of many, not only of the author. Therefore, we thank the thousands of students we had during the 38 years of teaching. They gave us the opportunity to test the content. Also, the book should not been written the way it is if not standing on the shoulders of those, who preceded it. Although we started our carrier as a structural engineer, our predecessor Professor Antoine de Grave planted the seeds that fed the interest in building physics. Bob Vos of TNO, the Netherlands, and Helmut Künzel of the Fraunhofer Institut für Bauphysik, Germany, showed the importance of experimental work and field testing to understand whole building and building part or assembly performance, while Lars Erik Nevander of Lund University, Sweden, taught that solving problems in building physics does not always demands complex modelling, mainly because reality in building construction is much more complex than any model can simulate.

During the four decades at the Unit of Building Physics and Sustainable Construction within the Department of Civil Engineering of the KULeuven, several researchers, then PhD students, got involved. They all contributed by the topics chosen to the advancement of the research done at the unit. Most grateful I am to Gerrit Vermeir, my colleague from the start in 1975, professor emeritus now, to Staf Roels, Dirk Saelens, Hans Janssen and Bert Blocken, who succeeded me as professors at the unit.

The experience gained the first 4 years of my career as a structural engineer and building site supervisor for a medium-sized architectural office, as building assessor during some 50 years, and as operating agent of four IEA, EXCO on Energy in Buildings and Communities Annexes forced me to rethink the engineering-based performance approach each time again. The many ideas exchanged in Canada and the United States with Kumar Kumaran of NRC, Paul Fazio of Concordia University in Montreal, Bill Brown, William B. Rose of the University of Illinois in Urbana-Champaign, Joe Lstiburek of the Building Science Corporation, Anton Ten Wolde and those participating in ASHRAE TC 1.12 'Moisture management in

buildings' and TC 4.4 'Building materials and building envelope performance' were also of great value.

Finally, I thank my family, my wife Lieve, who managed living together with a busy engineering professor, our three children, our children in law and our grand children.

March 2023
KU Leuven, Leuven, Belgium

Hugo S.L.C. Hens

About the Author

Dr. Ir. Hugo S.L.C. Hens is an emeritus professor of the University of Leuven (KU Leuven), Belgium. Until 1972, he worked as a structural engineer and site supervisor at a mid-sized architectural office. After the sudden death of his predecessor and promoter Professor A. de Grave in 1975 and after defending his PhD thesis, he stepwise built up the Unit of Building Physics at the Department of Civil Engineering.

He taught Building Physics from 1975 to 2003, performance-based building design from 1975 to 2005 and building services from 1975 to 1977 and 1990 to 2008. He authored and co-authored 68 peer-reviewed journal papers and 174 conference papers about the research done, has helped to manage hundreds of building damage cases and acted as coordinator of the CIB W40 working group on Heat and Mass Transfer in Buildings from 1983 to 1993. Between 1986 and 2008, he was operating agent of the Annexes 14, 24, 32 and 41 of the IEA EXCO on Energy in Buildings and Communities. He is a fellow of the American Society of Heating, Refrigeration and Air Conditioning Engineers (ASHRAE).

1

Heat Transfer

1.1 In General

1.1.1 Heat

1.1.1.1 What?

A first description of what heat is comes from thermodynamics, a discipline that considers everything as a system surrounded by an environment, with which energy is exchanged. Can be the system: a material, a building assembly, a building, a heating, ventilating, air Conditioning (HVAC) installation, even a whole city. Energy transmitted as work between both looks purposeful and organized, energy transmitted as heat diffuse and chaotic. A second description of what heat is resides in particle physics, where heat refers to the statistically distributed kinetic energy of atoms and free electrons. Whatever, heat is the least noble, most diffuse form of energy, to which each nobler form degrades, see the second law of thermodynamics.

1.1.1.2 Sensible Heat

Sensible heat is unambiguously linked to what people feel, the temperature, see below. Transferring sensible heat, be it by conduction, convection and radiation, demands differences in temperature.

Conduction refers to the heat flow in a medium induced by its vibrating atoms, whose spheres of influence collide, and the movement of free electrons. Heat transmitted between solids at different temperature in ideal contact with each other and between points in these is straightforwardly conduction-based. This is also the case between gases and liquids and contacting surfaces. According to the second law of thermodynamics, conduction always moves direction lower temperatures. It needs a medium and does not induce macroscopic movement.

Convection in turn is the result of macroscopic movement in liquids and gases, in which temperature differences exist or that touch colder or warmer surfaces. As well external forces, differences in density or both together are inducing such movements and fix the type of convection generated: forced, natural or mixed. Convection needs a medium.

Radiation finally concerns heat transferred between surfaces due to them emitting and absorbing electromagnetic waves. Above 0 K, each surface radiates. When two

or more surfaces at different temperature see each other, the result is heat exchanged between them. Radiation does not need a medium, while the laws governing it are very different from these shaping conduction and convection.

1.1.1.3 Latent Heat

Latent heat is linked to the changes of state between solid, liquid and gaseous. A release, for example liquid evaporating, or a deposit, for example liquid turning solid, require no differences in temperature, although both will impact the amounts of heat moving. Water evaporating absorbs the sensible heat of evaporation, so creates a heat sink. When then the water vapour so formed moves to a colder spot, where it condenses, the sensible heat of evaporation is emitted again, creating a heat source. Such sources and sinks not only impact the temperature profile in materials and assemblies, but they also have quite some impact on the sensible heat transferred.

1.1.2 Temperature

The temperature, described above as what people feel, mirrors the heat quality. Higher temperatures stay for more quality. Thanks to the related increase in kinetic energy of atoms and free electrons, the result is more exergy, which represents the potential to convert more heat into work via a cyclic process. Instead, lower temperatures and related decrease in the kinetic energy of atoms and free electrons stay for less exergy. Heightening the temperature of a system requires warming it, lowering the temperature of a system cooling it. Like any potential, temperature is a scalar. Sensing it is not a problem but measuring it is. Happily, many material properties depend on temperature, which allows its indirect quantification. A mercury thermometer allows scaling by using the volumetric expansion of mercury when heated and its volumetric contraction when cooled. In a Pt100 thermometer, the change of the electrical resistance of a platinum wire with temperature is used, while for thermocouples the varying contact potential between two metals is.

The SI system uses two temperature scales, one empiric, the degree Celsius ($^{\circ}\text{C}$) with θ as symbol, and the other thermodynamic, the degree Kelvin (K) with T as symbol. 0°C coincides with the triple point of water, and 100°C with its boiling point at 1 Atmosphere. 0 K instead stands for the absolute zero, and 273.15 K for the triple point of water. Temperature differences are given in K, temperatures in $^{\circ}\text{C}$ or in K with as relation between both:

$$T = \theta + 273.15$$

Instead of degree Celsius ($^{\circ}\text{C}$), the USA still uses degree Fahrenheit ($^{\circ}\text{F}$). The link between the two is:

$$^{\circ}\text{F} = 32 + 9/5^{\circ}\text{C}$$

1.1.3 Why are Heat and Temperature so Compelling?

That heat is an issue in buildings, follows from the human demand for thermal comfort. In cold and temperate climates, the comfort temperatures required demand

heating during the colder months. In most cases, the heat sources still used are oil and natural gas. Their overall share in delivering the end energy needed causes such CO₂ release, that since the second half of the 1970s, energy efficiency became imperative. A prime opportunity to realize is by minimizing the heat loss through the building envelope. Knowing which envelope properties have a decisive influence on that, became a necessity for designers and builders.

Which temperatures are considered as being important depends on the situation and possible consequences. In winter, inside surface temperatures close to the air temperature indoors felt as comfortable upgrade thermal comfort. Instead, those much lower not only degrade the thermal comfort but also increase mould and surface condensation risk, both perceived as triggering healthiness. Also, too high summer temperatures indoors clash with thermal comfort and IAQ, while high temperature differences across outer assemblies increase the air and moisture movement in them, the thermal stresses experienced and crack risk. Large temperature gradients also favour the displacement of dissolved salts, while high temperatures accelerate the chemical breakdown of synthetics. Furthermore, too many temperature fluctuations from above to below freezing may damage wet, frost-sensitive porous materials. Whether all these effects will remain controllable, depends on how building assemblies are designed and built.

1.1.4 Some Definitions

Amount of heat, symbol Q , unit [J]

Quantifies the energy exchanged as heat. As heat is a scalar, the amount also is.

Heat flow, symbol Φ , unit [J/s] = [W]

Stands for the heat exchanged per unit of time. Heat flow is a measure for 'power', thus, a scalar.

Heat flux, symbol \mathbf{q} , unit [W/m²]

Quantifies the heat exchanged per unit of time through a unit surface normal to the flow direction. The flux so is a vector with same direction as the surface vector. Its components in Cartesian coordinates are q_x, q_y, q_z , in polar coordinates q_R, q_ϕ, q_θ .

Solving a heat transfer problem now means determining the scalar temperature field (T) and the vectorial heat fluxes field (\mathbf{q}). Computing both so requires a scalar and a vector equation.

1.2 Conduction

1.2.1 Conservation of Energy

A first relation between the heat flux (\mathbf{q}) and temperature (T) follows from the conservation of energy axiom. In case an infinitely small material volume is the system and what is around the environment, then, without mass displacement, the energy balance between both writes as:

$$d\Phi + d\Psi = dU + dW \quad (1.1)$$

with $d\Phi$ the resulting heat flow between system and environment, $d\Psi$ the heat dissipated uniformly in the system, dU the change in the system's internal energy and dW the labour exchanged with the environment, all per unit of time. Dissipation could include the heat produced by an exothermic reaction, the heat absorbed by an endothermic reaction, the Joule effect by an electric current passing through, latent heat released or absorbed, etc. The labour exchanged equals:

$$dW = Pd(dV) = P d^2V$$

with P the pressure exerted in Pa. The conservation balance so states that the heat exchanged ($=d\Phi$), released or absorbed, modifies the internal energy in this infinitely small material volume, while causing a labour exchange with the environment. If isobaric, the balance reshuffles to

$$d(U + PdV) = dQ + dE$$

with $U + PdV$ the enthalpy (H). The resulting heat flow, the change in enthalpy and the heat dissipated now write as:

$$d\Phi = -\text{div}(\mathbf{q})dV \quad dH = \left| \frac{\partial(\rho c_p T)}{\partial t} \right| dV \quad d\Psi = \Phi' dV$$

with c_p the specific heat capacity at constant pressure of the material (J/(kg·K)), ρ its density (kg/m³) and Φ' the heat dissipated per unit of time and volume, positive if a source, negative if a sink. The three turn the conservation equation into:

$$\left(\text{div}(\mathbf{q}) + \Phi' + \frac{\partial(\rho c_p T)}{\partial t} \right) dV = 0 \quad (1.2)$$

For solids and liquids, the specific heat capacity hardly depends on the change of state. So, one value, symbol c , can be used with the product ρc equal to the volumetric specific heat capacity. For gases, the value varies with the change of state, giving as relation between the specific heat capacity at constant pressure (c_p) and at constant volume (c_v):

$$c_p = c_v + R$$

with R the specific gas constant (in Pa·m³/(kg·K)). Because conservation of energy now holds for any infinitely small material volume, the relation between heat flux (\mathbf{q}) and temperature (T) so becomes:

$$\text{div } \mathbf{q} = -\frac{\partial(\rho c T)}{\partial t} - \Phi' \quad (1.3)$$

1.2.2 Conduction Laws

1.2.2.1 First Law

The name 'first law' is given to the empirical vector equation between heat flux and temperature, advanced by the French physicist Fourier:

$$\mathbf{q} = -\lambda \mathbf{grad } T = -\lambda \mathbf{grad } \theta \quad (1.4)$$

It states that the conductive heat flux in a point somewhere in a solid, liquid or gas varies proportionally to the temperature gradient there. The proportionality value λ figures as a material property called the ‘thermal conductivity’ with as units $W/(m \cdot K)$. It expresses the ability of a medium to conduct heat. The minus sign indicates that the flux and the temperature gradient, which as vector goes from colder to warmer, oppose each other. Thermodynamics in fact learns that, if not forced externally, heat always moves direction colder. Otherwise, the entropy would decrease without energy input, which is impossible. Following observation supports the first law. With the surfaces of equal temperature, called isotherms, drawn in a construction detail and the heat fluxes visualized by tracing the lines of equal flux, called isoflux lines, seen is that the last develop perpendicular to the isotherms, come closer and break up more where the isotherms do (Figure 1.1).

At the same time, in each material, the fluxes remain proportional to their thermal conductivity, which is often assumed to be a scalar and constant, even though for building and insulating materials its value depends on temperature, moisture content, sometimes the material’s thickness and its age, while for anisotropic materials it becomes a tensor. Often, the term ‘apparent’ is therefore added to the supposedly constant value.

In right-angled Cartesian coordinates $[x, y, z]$, the heat flux along the three axes writes as:

$$q_x = \mathbf{q}_x \mathbf{u}_x = -\lambda \frac{\partial T}{\partial x} \quad q_y = \mathbf{q}_y \mathbf{u}_y = -\lambda \frac{\partial T}{\partial y} \quad q_z = \mathbf{q}_z \mathbf{u}_z = -\lambda \frac{\partial T}{\partial z}$$

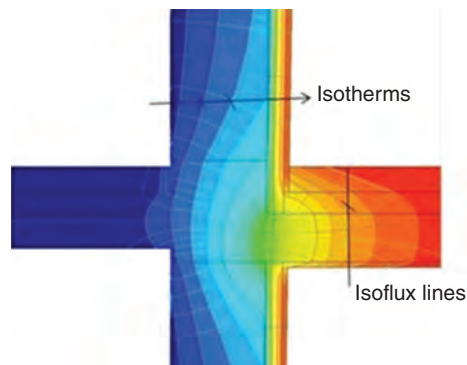
In case $^{\circ}C$ instead of K is used, the heat flow across a surface dA with direction n equals:

$$d\Phi_n = \mathbf{q} d\mathbf{A}_n = -\lambda \frac{\partial \theta}{\partial n} dA_n u_n^2 = -\lambda \frac{\partial \theta}{\partial n} dA_n$$

Along each of the three axes, this gives as heat flow:

$$d\Phi_x = -\lambda \frac{\partial \theta}{\partial X} dA_x \quad d\Phi_y = -\lambda \frac{\partial \theta}{\partial y} dA_y \quad d\Phi_z = -\lambda \frac{\partial \theta}{\partial Z} dA_z$$

Figure 1.1 Floor with balcony traversing an outer wall insulated inside, lines of equal temperature and equal heat flux (the isotherms and isoflux lines).



1.2.2.2 Second Law

The second law is embedding the conduction equation in the conservation of energy one:

$$\operatorname{div}(\lambda \mathbf{grad} T) = \frac{\partial(\rho c T)}{\partial t} - \Phi' \quad (1.5)$$

This scalar relation allows calculating temperature fields. In case the thermal conductivity and the volumetric specific heat capacity are constant, the equation simplifies to what is called Fourier's second law:

$$\nabla^2 T = \left(\frac{\rho c}{\lambda} \right) \frac{\partial T}{\partial t} - \frac{\Phi'}{\lambda} \quad (1.6)$$

with ∇^2 the Laplace operator, in Cartesian coordinates equal to:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad (1.7)$$

After a deeper digging in what the thermal conductivity represents at the material level, further discussion will focus on applying both laws to building assemblies.

1.2.3 Thermal Conductivity

1.2.3.1 In General

As mentioned, the property λ is not a constant. Its value in fact depends on the way heat is transferred in a material, which causes the already mentioned dependence on temperature, moisture content, sometimes thickness and age. This demands a closer look to how λ gets a value.

1.2.3.2 Heat Transfer Modes Fixing the Property

In dry porous materials four modes coexist (Figure 1.2): conduction along the material matrix, conduction through the pore gas, convection in the pore gas and radiation in each pore between its walls. When humid, two modes add: conduction in the water film against the pore walls and a latent heat exchange by evaporation at the warmer part of these walls, diffusion of the vapour so formed through the pore

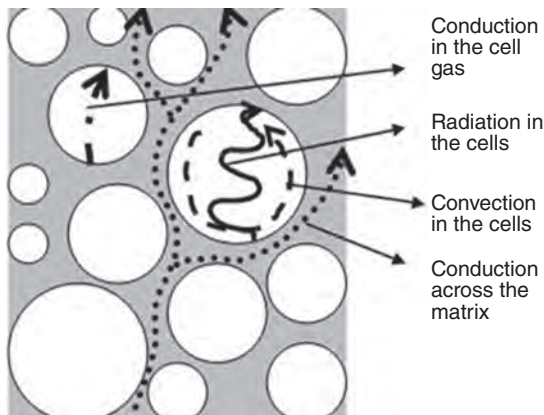


Figure 1.2 Heat flow modes in a porous material.

and condensation against the colder part of the pore walls. The value of a material's thermal conductivity depends on factors linked to these four or six modes.

If only conduction along the matrix and through the pore gas intervened, the thermal conductivity should equal:

$$\lambda_c = \lambda_M(1 - \Psi) + \lambda_G \frac{2\Psi}{1 + \Psi} \quad (1.8)$$

with λ_M the thermal conductivity of the matrix material, λ_G the thermal conductivity of the gas in the pores and Ψ the material's total porosity, equal to:

$$\Psi = (\rho_s - \rho) / \rho_s \quad (1.9)$$

with ρ the density and ρ_s the specific density of the matrix material, both in kg/m^3 . Accordingly, the thermal conductivity should decrease the higher the total porosity. Or, a more porous material with lower density (ρ), must see its thermal conductivity drop, a fact experiments prove, see Figure 1.3. Material matrixes with lower thermal conductivity or gases in the pores insulating better than stagnant air should show a same trend. If pores could be vacuumed, the gas-related thermal conductivity should even turn zero ($\lambda_G \approx 0$).

In case other gases than air are filling the pores of a highly porous material, diffusion of air in, diffusion of part of these gases out or of being adsorbed by the material matrix will slowly increase the λ -value. In the first weeks after manufacturing, this increase obeys:

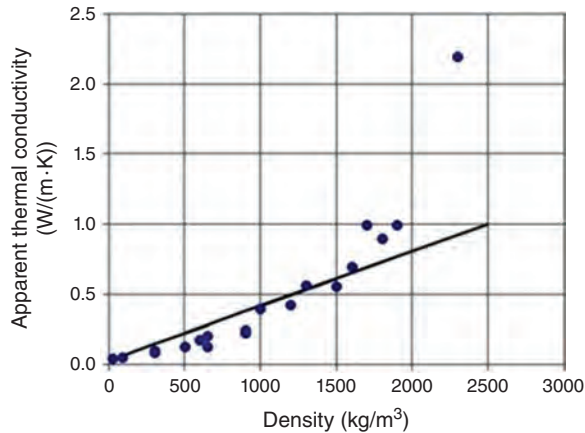
$$\lambda_c = \lambda_c(0) + C_1 \sqrt{t}$$

where initially C_1 changes inversely proportional to the diffusion resistance factor μ of the matrix material and proportionally to the temperature with exponent $n < 1$ (T^n , T in K). Later the increase slows down to:

$$\lambda_c = \lambda_c(0) + (\lambda_c(\infty) - \lambda_c(0))[1 - \exp(-C_2 t)]$$

with C_2 depending on the temperature and the diffusion resistance factor μ of the matrix material the same way as C_1 does, be it with values slackening the pace. Or, to

Figure 1.3 Thermal conductivity versus density. The full line stands for glass, from cellular to massive, $\lambda_M = 1 \text{ W/(m}\cdot\text{K)}$, the dots for measured values for other materials.



more or less permanently store another gas than air in the pores, the matrix should be as vapour retarding as possible. An alternative for thermally insulating materials consists of covering the fresh boards with a vapour-tight lining. Or, to act as thermal insulation, materials need a very low density with, if the matrix is enough vapour retarding, a gas filling the pores that retards heat better than air does.

However, in larger pores, convection may develop. Its impact is quantified by multiplying the thermal conductivity of the pore gas with its convection-related Nusselt number ($X \geq 1$):

$$\lambda_c = \lambda_M(1 - \Psi) + X\lambda_G \frac{2\Psi}{1 + \Psi} \quad (1.10)$$

As a result, more heat will flow through the material than without convection. Or, looking to insulation materials, their pores should be small enough to keep the Nusselt number 1. Thus, an insulation material should not only be very porous, but it should also have small enough pores.

But additionally, in a cell where the walls that are not perfectly reflective and at different temperature radiant heat is exchanged giving a radiant term (λ_R) completing the λ_c -value just given:

$$\lambda = \lambda_m(1 - \Psi) + X\lambda_G \frac{2\Psi}{1 + \Psi} + F_{RC} \underbrace{\frac{4C_b T_m^3 d}{100^4 \left(\frac{1}{e_1} + \frac{1}{e_2} + n \frac{1 + \rho - \tau}{1 - \rho + \tau} - 1 \right)}}_{\lambda_R} \quad (1.11)$$

with:

$$F_{RC} = 1 + 100^4 \frac{\lambda_c(1/e_1 + 1/e_2 - 1)}{4C_b T_m^3 d} \left[\frac{\Delta\theta_1 + \Delta\theta_n}{\Delta\theta/n} - 1 \right]$$

In case of insulation materials, in F_{RC} , the factor that corrects the thermal conductivity for the interaction between radiation, convection and conduction, is n standing for the number of pore walls, assumed parallel to the board's two faces, filling its thickness d . $\Delta\theta$ is the temperature difference between both faces, while $\Delta\theta_1$ and $\Delta\theta_n$ are the temperature differences between the linings that may cover both faces and the first pore wall encountered. In the λ_R formula, ρ and τ are the long wave reflectivity and transmissivity of these parallel pore walls, while for thermal insulation materials e_1 and e_2 are the long wave emissivity, side material of the linings covering both faces. As the radiant term λ_R is proportional to the third power of the temperature and the thermal conductivities of the matrix and the pore gas are temperature dependent, the overall relation with temperature may be rewritten as:

$$\lambda = \lambda_o + a_1\theta^n + a_2\theta^3 \quad (1.12)$$

with $0 < n < 1$. Between -20 and 50°C , that equation becomes \pm linear:

$$\lambda = \lambda_o + a_R\theta = \lambda_o (1 + a'_R\theta) \quad (1.13)$$

The lighter a highly porous material, meaning the thinner the pore walls or the larger the pores, the higher a_R and the more temperature dependant its thermal conductivity. Because few large than small pores fit into a given layer thickness and

because thin pore walls show a higher long wave transmissivity than thicker ones, radiation gains importance in equally heavy materials having as well small or large pores. But the thermal conductivity must increase with a board's thickness. In fact, with the ratio between thickness (d) and mean pore width (d_p) replacing the number of pore walls, thickness appears in the numerator of λ_R but stays hidden in the denominator. This way, the radiation term (λ_R) shifts from 0 for a zero thickness to an asymptotic value ($\lambda_{R\infty}$) when infinitely thick:

$$\lambda_{R\infty} \approx \frac{4FC_b T_m^3 d_P}{100^4 \left[\frac{1 + \rho - \tau}{1 - \rho + \tau} \right]} \tag{1.14}$$

That asymptote increases for larger pores and pore walls transmitting more radiation, thus for materials with lower density. Or, radiant exchange prevents the thermal conductivity to continually drop with lower density. In fact, once below a certain value, further decrease demands larger pores or thinner pore walls. In both cases, radiation increases, turning the thermal conductivity into a sum of a monotonously decreasing conductive and an increasing radiant part. Or, surely for thermally insulating materials with given thickness, the λ -value must at some density pass a minimum (Figure 1.4):

$$\lambda = b_1 + b_2\rho + b_3/\rho \tag{1.15}$$

In a moist material, heat conduction in the adsorbed water layers and condensed water islands adds. For open porous materials, the result is a linear relation between thermal conductivity and moisture ratio, be it that for very porous insulating materials, the relation, now with the volumetric moisture ratio, is rather parabolic:

$$\lambda = \lambda_d(1 + a_X X) \quad \lambda = \lambda_d(1 + a_\Psi \Psi + b_\Psi \Psi^2) \tag{1.16}$$

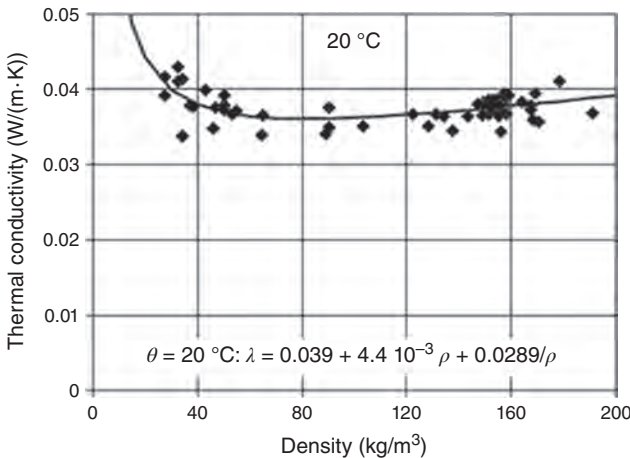


Figure 1.4 Mineral wool, thermal conductivity versus density.

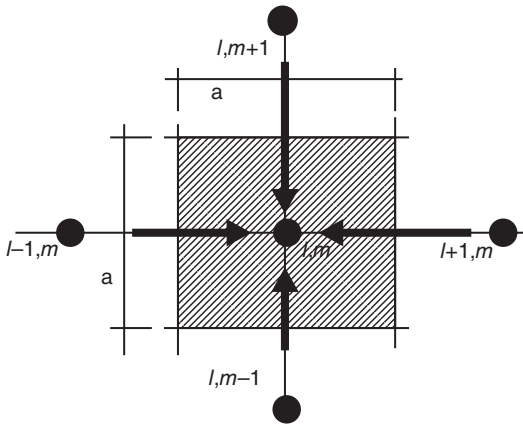


Figure 2.8 Conservation of mass: sum of airflows from the nearest surrounding to a central control volume zero.

In it, A is the contact surface between central and adjacent control volume and a the distance separating their centres (Figure 2.8).

For p control volumes, the result is a system of p equations with p unknowns:

$$[K'_a]_{p,p} [P_a]_p = [K'_{a,i,j,k} P_{a,i,j,k}]_p \tag{2.26}$$

In it, $[K'_a]_{p,p}$ is a p rows, p columns permeance matrix, $[P_a]_p$ a column matrix with the p unknown air pressures and $[K'_{a,i,j,k} P_{a,i,j,k}]_p$ a column matrix with all known air pressures. Once the unknowns are solved, the airflow exchanged between adjacent and central control volumes follows from:

$$G_{a,i,j,k} = K'_{a,i,i+j} (P_{a,i+j} - P_{a,i}) \tag{2.27}$$

For anisotropic materials, a same algorithm applies on condition each of the lines linking the centres of adjacent to central control volumes coincides with the main directions of the permeability tensor. Related $(K'_a)_{z,y,z}$ -values can then be used.

Under non-isothermal conditions, per control volume thermal stack couples the air to the heat balances. Most of the time, solving requires iteration between both.

2.2.5 Airflow Through Assemblies with Air-open Layers, Leaky Joints, Leaks, Cavities, etc.

For the flow equations, see above. Most assemblies combine air-open layers, open porous materials, joints and cavities. In such case, writing the conservation law as a partial differential equation does not work. An approach exists of transforming the assembly into an equivalent hydraulic circuit composed of well-chosen points, connected by air permeances (Figure 2.9). Per point, the sum of the airflows coming from the adjacent points must be 0. As each flow can be written as $K^x_a \Delta P_a$, insertion in the conservation law gives:

$$\sum_{\substack{i=l,m,n \\ j=\pm 1}} (K^x_{a,i+j} P_{a,i+j}) - P_{a,l,m,n} \sum_{\substack{i=l,m,n \\ j=\pm 1}} K^x_{a,i+j} = 0 \tag{2.28}$$

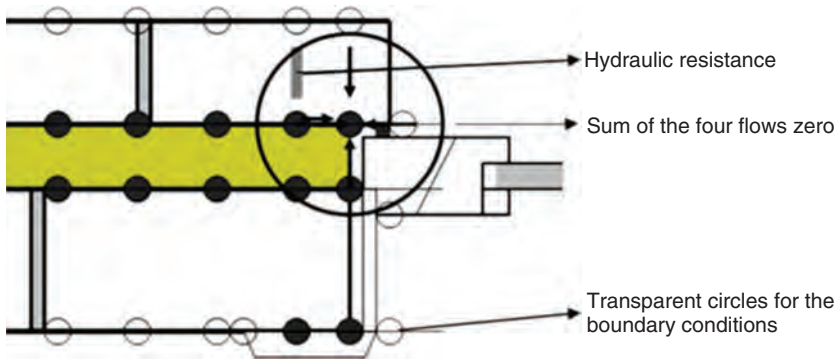


Figure 2.9 Equivalent hydraulic circuit.

A total of p points so results in a system of p non-linear equations with p unknown air pressures. If the boundary conditions are known, solving gives these unknowns. Insertion into the flow equations then ensues the airflows between the control volumes. Of course, non-linearity requires iteration, starting by assuming values for the p unknown air pressures, which allows to calculate the related air permeances as function of these and solve the system. Next, all permeances are recalculated using the p air pressures just found and the system is solved again. Iterating so should continue till the deviation between the preceding and new results dives below a pre-defined number.

In case the flow through a flat assembly, composed of air open layers, is by exception one-dimensional, a same air flux g_a must pass each layer with non-linear air permeance:

$$\begin{aligned}
 \text{Layer 1} \quad g_a &= K_{a_1} \Delta P_{a_1} = a_1 \Delta P_{a_1}^{b_1} \quad \text{or} \quad \left(\frac{g_a}{a_1} \right)^{1/b_1} = \Delta P_{a_1} \\
 \text{Layer 2} \quad g_a &= K_{a_2} \Delta P_{a_2} = a_2 \Delta P_{a_2}^{b_2} \quad \text{or} \quad \left(\frac{g_a}{a_2} \right)^{1/b_2} = \Delta P_{a_2} \\
 \text{Layer } n \quad g_a &= K_{a_n} \Delta P_{a_n} = a_n \Delta P_{a_n}^{b_n} \quad \text{or} \quad \left(\frac{g_a}{a_n} \right)^{1/b_n} = \Delta P_{a_n} \\
 \text{Sum:} \quad g_a &\left[\sum \frac{g_a^{1/b_i - 1}}{a_i^{1/b_i}} \right] = \Delta P_a \quad (2.29)
 \end{aligned}$$

ΔP_a is the air pressure differential between both end faces. Solving requires again iteration. Once the 'correct' air flux is known, the air pressure course in the assembly then follows from the layer equations.

2.2.6 Airflow at the Building Level

2.2.6.1 Definitions

In buildings, air moving between spaces and to and from outdoors is called inter-zonal flow, air movement in a space intrazonal flow. Questions which intrazonal

must answer include: What with air looping? How do the zone and ventilation air mix? Which corners lack air washing? Related calculations require Computerized Fluid Dynamics (CFD), not discussed here. Only interzonal flow is. Calculating starts with replacing spaces by nodes and fixing the flow paths between these.

2.2.6.2 Thermal Stack

With the outdoors as reference, stack writes as:

$$P_a - g\rho_a z = P_a - z \frac{gP_a}{R_a} \left(\frac{1}{T_e} - \frac{1}{T_i} \right) \approx P_a - 3460z \left(\frac{1}{T_e} - \frac{1}{T_i} \right) \quad (2.30)$$

with T_i the temperature in the space in K, T_e the temperature outdoors in K and z the height considered in m. Between equally warm spaces at different height coupled by a flow path, the pressure difference with outdoors is:

$$\Delta_{12}(P_a - \rho g z) = P_{a1} - P_{a2} - 3460(z_1 - z_2) \left(\frac{1}{T_e} - \frac{1}{T_i} \right)$$

with z_1 and z_2 the height of the two zonal points above a horizontal reference plane. Both spaces at different temperature ($T_{i,1}$, $T_{i,2}$) changes this stack equation into:

$$\begin{aligned} \Delta_{12}(P_a - \rho g z) = & P_{a1} - P_{a2} - 3460 \\ & \times \left[(z_o - z_1) \left(\frac{1}{T_e} - \frac{1}{T_{i,1}} \right) + (z_2 - z_o) \left(\frac{1}{T_e} - \frac{1}{T_{i,2}} \right) \right] \end{aligned}$$

with z_o the height of the horizontal plane (o) where temperature $T_{i,1}$ becomes $T_{i,2}$.

2.2.6.3 Large Openings

The airflow through a closed window is:

$$G_a = a L \Delta P_a^{2/3}$$

with a the air permeance per running metre of joints in the casement in $\text{kg}/(\text{m}\cdot\text{s}\cdot\text{Pa}^{2/3})$, L their length in m, ΔP_a the pressure differential between the zone node and either outdoors or at the zone node on the other side of the window, then part of a partition. Open, doors and windows instead act as large openings with an air permeance (K_a) depending on the driving force. Where wind and fans give a uniform flow through, stack activates two equally large flows, one up from warm to cold and an equal one down from cold to warm, with the dividing line in the middle of the opening (Figure 2.10).

The airflow in case wind or a fan care for the pressure differences equals:

$$G_a = C_f B H \sqrt{2\rho_a \Delta P_a} \quad (\text{kg/s}) \quad (2.31)$$

giving as air permeance:

$$K_a = (C_f B H \sqrt{2\rho_a}) \Delta P_a^{-0.5} \quad (2.32)$$

with C_f a flow factor with value 0.33–0.7, B the width and H the height of the door or window. With thermal stack, the flows in to out and out to in become:

$$G_{a1} = -G_{a2} = \frac{C'_f B}{3} \sqrt{\frac{\rho_a g P_a H^3}{R_a} \left(\frac{1}{T_1} - \frac{1}{T_2} \right)} \quad (\text{kg/s}) \quad (2.33)$$

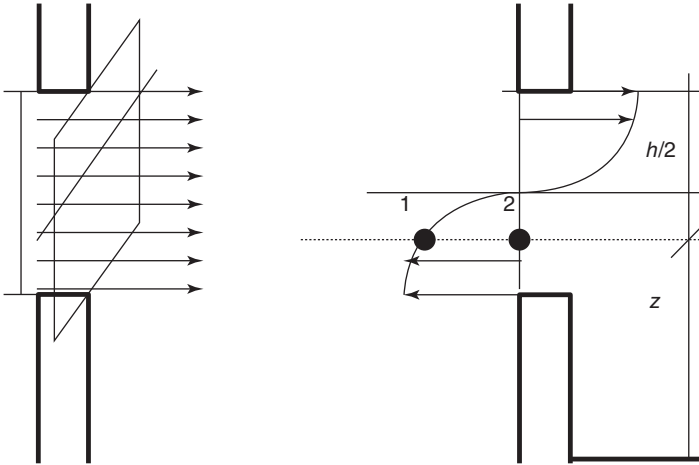


Figure 2.10 Large opening, uniform flow by wind and fans and stack-induced flow.

giving as air permeance $\left(\Delta p_{T,\max} = gP_a H/R_a (T_e^{-1} - T_2^{-1} - T_e^{-1} + T_1^{-1}) \approx 3450 H (T_1^{-1} - T_2^{-1})\right)$:

$$K_{a1} = -K_{a2} = \frac{C'_f B H \sqrt{\rho_a}}{3} \left[3450 H \left(\frac{1}{T_1} - \frac{1}{T_2} \right) \right]^{-0.5} \quad (2.34)$$

In it, R_a is the gas constant of air and C'_f the flow factor characterising the two-way move. The velocity of the air in- and outgoing along the opening's height is parabolic, a consequence of Bernoulli's law applied to a flow path at height z above the dividing line:

$$\rho_{a1} g z = \frac{\rho_{a2} v_{a,z}^2}{2} + \rho_{a2} g z$$

This gives as velocity:

$$v_{a,z} = \sqrt{2gz \left(\frac{\rho_{a1} - \rho_{a2}}{\rho_{a2}} \right)}$$

The air permeance equation given so follows from first integrating the air speed over half the height of the opening, then using the ideal gas law to convert air density into the related temperature and finally by multiplying the result with the flow factor.

2.2.6.4 The Conservation Law Applied

Buildings include several spaces, thus, several nodes, see Figure 2.11.

Per node, the algebraic sum of the n airflows coming in and m going out must be 0, or:

$$\sum_{i=1}^{m+n} G_{a,i} = 0 \quad (2.35)$$

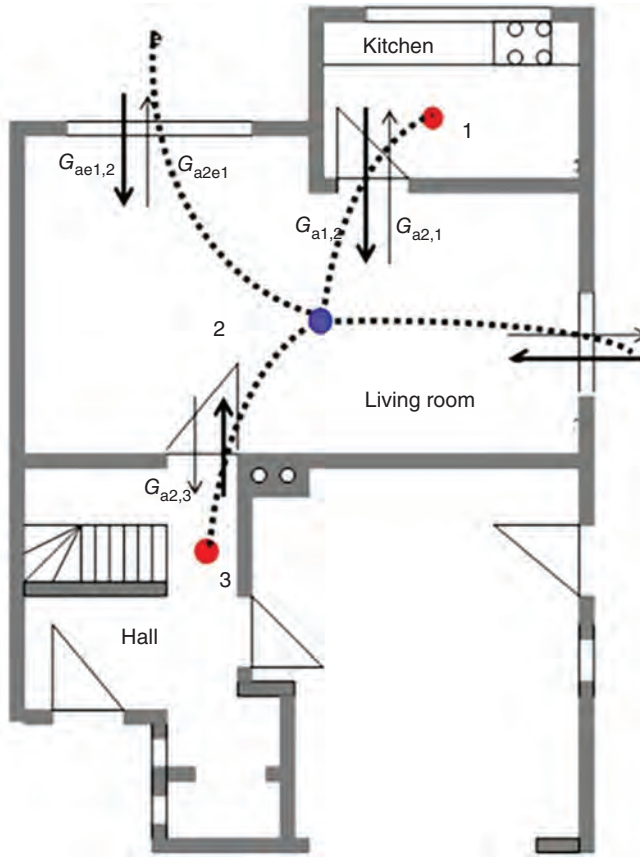


Figure 2.11 The ground floor of the building considered.

The living room, zone 2 in Figure 2.11, has two windows and two doors, one to the kitchen, zone 1, one to the hall, zone 3. All partitions and opaque outer walls are now assumed airtight, which in reality may not be the case. The building has no fan-driven ventilation and the temperatures in- and out are set equal. Wind then is the only driving force. Air will enter or leave the living room through the operable window sashes ($G_{a,2-e1}$, $G_{a,2-e2}$) and through both doors ($G_{a,2-1}$, $G_{a,2-3}$). Wind pressures outdoors are known. The four flows supposed to move inwards equal:

$$G_{a,2-e1} = K_{a,2-e1}(P_{a,e1} - P_{a,2}) \quad G_{a,2-e2} = K_{a,2-e2}(P_{a,e2} - P_{a,2})$$

$$G_{a,2-1} = K_{a,2-1}(P_{a,1} - P_{a,2}) \quad G_{a,2-3} = K_{a,2-3}(P_{a,3} - P_{a,2})$$

Summing and reshuffling give:

$$-(K_{a,2-e1} + K_{a,2-e2} + K_{a,2-1} + K_{a,2-3})P_{a,2} + K_{a,2-e1}P_{a,e1} + K_{a,2-e2}P_{a,e2} + K_{a,2-1}P_{a,1} + K_{a,2-3}P_{a,3} = 0$$

or:

$$-P_{a,2} \sum (K_{a,i-j/e}) + \sum (K_{a,i-j} P_{a,j}) = - \sum (K_{a,i-e} P_{a,e})$$

The three unknowns in this equation are the air pressure in the living room ($P_{a,2}$), the kitchen ($P_{a,1}$) and the hall ($P_{a,3}$).

Besides living room, kitchen and hall, the ground floor also consists of a garage and a toilet, while the staircase links the ground floor to three bedrooms and a bathroom on the first floor. For each, the flows in and out, the last supposed inward, give sum 0. The result is as many equations as there are spaces. In matrix form:

$$[K_a]_{n,n} [P_a]_n = [K_{a,e} P_{a,e}]_n \quad (2.36)$$

If non-linear, solving that system demands iteration.

Is the building warmer or colder than outdoors, all leaks must be located. The spaces then get nodes at the height of each leak, after which thermal stack with outdoors against a reference height complements the pressure differences between adjacent nodes at different height. Between nodes one above the other in a same space, they get coupled by very large fictitious air permeances. With the temperature in all spaces and outdoors known, stack figures as a known term in the balance equations. The temperature in unheated spaces, however, depends on the transmission losses, the internal gains and the temperature of the entering air. This obliges to combine mass and energy conservation with the correct temperatures following from assuming a value and alternatively solving the heat and air balances using the temperatures and airflows given by the preceding iteration, until the differences between new and preceding drop below a preset value. Is ventilation fan-driven, these act as sources with known pressure/flow characteristics.

2.2.6.5 Applications

Airflow through a narrow horizontal opening in a façade wall of an otherwise airtight space, take a trickle vent above a window, is hardly possible. Vertically opening windows set ajar instead care for air exchange, not only by thermal stack giving a twinned flow but also by pulsating wind pushing air in and out. Openings in two opposite walls enclosing a space in turn act in series. Has the one an air permeance $K_{a,1}$, the other an air permeance $K_{a,2}$, is $P_{a,1}$ the air pressure in front of the one, $P_{a,2}$ the air pressure beyond the other, then, if isothermal, the air balance is (Figure 2.12):

$$-(K_{a,1} + K_{a,2})P_{a,x} + K_{a,1}P_{a,1} + K_{a,2}P_{a,2} = 0$$

with $P_{a,x}$ the unknown air pressure in the space.

For air to flow, $P_{a,1}$ and $P_{a,2}$ must be different, which, if windy, is likely for openings in opposite outer walls. Of course, given the wind pressure increases with height, two openings at different height in a wall also act as a series circuit. How large the airflow will be, depends on their air permeances. If constants, the mass balance gives:

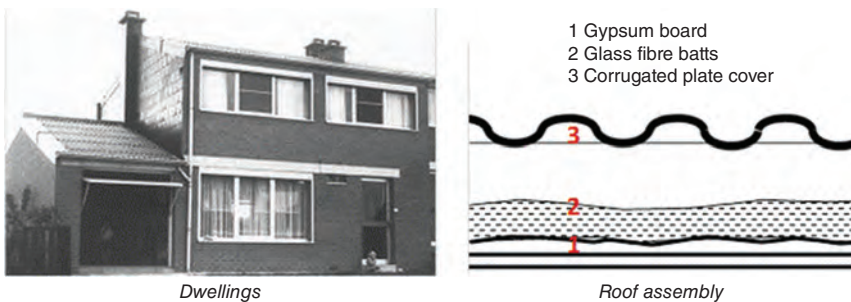
$$G_{a,1} = -G_{a,2} = G_a = K_{a,1}(P_{a,1} - P_{a,x}) = \frac{1}{\frac{1}{K_{a,1}} + \frac{1}{K_{a,2}}}(P_{a,1} - P_{a,2})$$

so reducing the ventilation rate to a 0.2 ach infiltration. A steep decrease in vapour pressure is seen, followed by a very slow climb to an equilibrium with the hygric memory of the room.

In reality, the increase after closing the window will go faster, mainly because the sorption active thickness underestimates the early, quick vapour release by diffusion. Or, a short peak ventilation after hours of usage has less effect than expected.

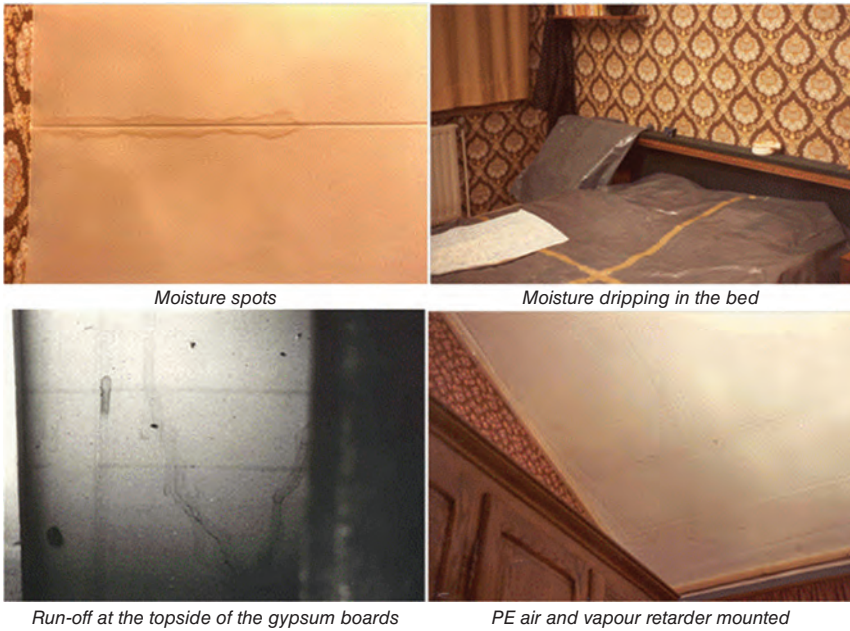
Problems and Solutions

Problem 31 A social estate consists of 24 two-story two-family houses with pitched roofs:



The 2.5 m high ground floor and equally high first-floor measure $7.2 \times 7.2 \text{ m}^2$. The long pitch of the asymmetric roof has a 17° , the short pitch a 10° slope. An open staircase connects both floors. No house has a purpose-designed ventilation system. Instead, air leakage and window opening have to guarantee an acceptable indoor air quality. The only difference between the 24 is the orientation: 4 have a front looking northwest, 2 a front looking north-west, 8 a front looking northeast, 3 a front looking east, 3 a front looking southeast and 4 a front looking southwest.

A few years after occupation, 41 houses had moisture spots spread over the ceiling of the sleeping room, while tenants complained about moisture dripping in their bed after cold nights. Inspection showed that the gypsum board forming the ceilings were mounted with open joints, that the insulating glass fibre bats with bituminous paper backing were laid out with open joints between the rafters with the flanges not overlapping against their underside. The backside of the corrugated fibre cement plates cover, the rafters and the top of the gypsum boards showed abundant traces of water run-off, while the board's inside face suffered from discolouration along the open joints, see the pictures below.



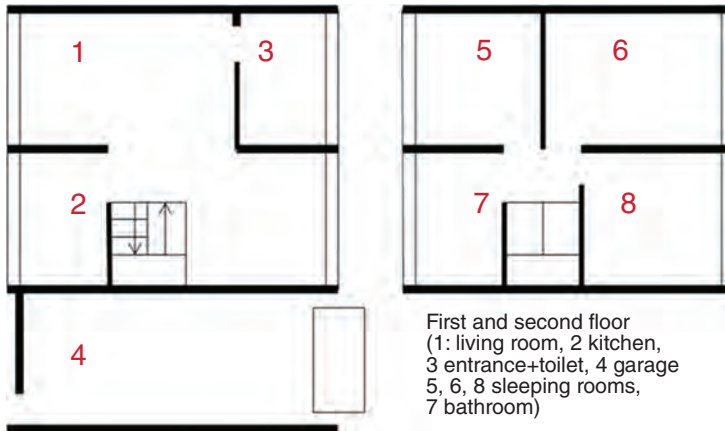
To clarify the randomness of the complaints, a correlation was sought between their severity and the average number of tenants. Looked was whether a cooking hood was present, how the annual end energy use for heating looked and which orientation the front façade had. Only the heating bill correlated with the severity of the complaints. On average, dwellings with severe ones consumed 128 GJ/a, and those with moderate ones 164 GJ/a. Tenants there either heated more rooms or ventilated more while heating.

During the winter 1981–1982, the inside temperature and RH were monitored in three dwellings, two with severe and one with moderate complaints (1 = moderate; 2 and 3 = severe). Before, in dwelling 2, a polyethene (PE) air and vapour retarder with all joints and overlaps carefully sealed was fixed against the underside of the gypsum board ceiling, see picture above. The table below summarizes the results.

	Parents' bedroom		Children bedroom		Bathroom	
	Temp., °C	Δp_{ie} , Pa	Temp., °C	Δp_{ie} , Pa	Temp., °C	Δp_{ie} , Pa
1	$13.6 + 0.42\theta_e$	$196 - 1.2\theta_e$	$14.1 + 0.42\theta_e$	$159 - 0.9\theta_e$		
2	$13.1 + 0.32\theta_e$	$373 - 14.7\theta_e$	$13.9 + 0.26\theta_e$	$237 - 2.5\theta_e$	$14.3 + 0.21\theta_e$	$457 - 17.7\theta_e$
3	$11.7 + 0.48\theta_e$	$324 - 10.8\theta_e$	$15.6 + 0.06\theta_e$	$411 - 34\theta_e$	$17.7 + 0.25\theta_e$	$395 - 19.4\theta_e$

The two with severe complaints showed the highest in- to outdoor vapour pressure difference; Happily, the PE-foil in dwelling 2 stopped the dripping completely.

Additional data



The volume out to out is 344.9 m^3 of which 149 m^3 for the ground floor. The enclosed air volume is 248.3 m^3 large, while the surface areas out to out of all envelope assemblies equal:

Opaque envelope assemblies	Area, m^2	Windows and outer door	Area, m^2
Floor on grade	51.8	Ground floor Front door	2.0
Cavity wall	Sidewall 1	Toilet	0.2
	Sidewall 2	Living room, front	5.4
	Front	Living room, back	5.4
	Rear	Kitchen	5.3
Roof	Large pitch	First floor Sleeping room 1	4.5
	Small pitch	Sleeping room 2	4.5
		Sleeping room 3	4.5
		Bathroom	1.8

The thermal transmittances and air permeances as built are:

Part	U-value, $\text{W}/(\text{m}^2 \cdot \text{K})$	K_a , $\text{kg}/(\text{m}^2 \cdot \text{Pa} \cdot \text{s})$
Façade: non-insulated cavity wall, plastered inside	1.66	0
Roof	0.49	$3.3 \cdot 10^{-4} \Delta P_a^{-0.33}$
Floor on grade	0.70	0
Window between both roof pitches	3.34	0
Glazing Double at the ground floor	2.70	
Single at the first floor	5.70	
Aluminium frames (20% of the window area)	5.90	

Overall air leakage equals $\dot{V}_a = a\Delta P_a^{0.67}$ (m^3/s), while the roof has a section

Layer (all air permeable)	d, m	λ , W/(m.K)	R, $\text{m}^2\cdot\text{K}/\text{W}$	μd , –
Corrugated plates	0.006	0.95		0.34
Air space	0.18		0.17	0.00
Thermal insulation	0.06	0.04		0.07
Vapour retarder	–	–		2.30
Air space	0.04		0.17	0.00
Gypsum board	0.0095	0.21		0.12

The inside surface film coefficient is $7.7 \text{ W}/(\text{m}^2\cdot\text{K})$ for the outer walls, $6 \text{ W}/(\text{m}^2\cdot\text{K})$ for the floor on grade and $10 \text{ W}/(\text{m}^2\cdot\text{K})$ for the roof. Outside, the values are $25 \text{ W}/(\text{m}^2\cdot\text{K})$ for the outer walls and floor on grade and $17 \text{ W}/(\text{m}^2\cdot\text{K})$ for the roof. The inside surface film coefficient for diffusion equals $2.6 \cdot 10^{-8} \text{ s}/\text{m}$.

The temperature indoors is 18°C and the air change rate at 50 Pa (n_{50}) 10.4 h^{-1} with the front and rear facade of the ground and first floor equally air permeable. Vapour release indoors touche $\approx 13.5 \text{ kg}/\text{day}$, while the outdoor climate during the cold week considered gives:

Temperature $^\circ\text{C}$	Temperature roof, $^\circ\text{C}$	Relative humidity for -2.5°C , %	Mean wind speed, m/s	Wind direction
-2.5	-3.9	98	3.8	NE

The mean wind speed is the one measured in the nearest weather station at a height of 10 m. As the estate forms a closed landscape, the effective terrain roughness of 1 m and a friction velocity of $0.47 \text{ m}/\text{s}$ changes this value into:

$$v = 1.12 \ln(h + 1) \text{ m}/\text{s}$$

with h height above grade. Wind pressure follows from $0.6Cv^2$ with C equal to:

	Wind angle with the normal to the front facade							
	0	45	90	135	180	225	270	315
Front	0.2	0.05	-0.25	-0.3	-0.25	-0.3	-0.25	0.05
Back	-0.25	-0.3	-0.25	0.05	0.2	0.05	-0.25	-0.3
Side left	-0.25	0.05	0.2	0.05	-0.25	-0.3	-0.25	-0.3
Side right	-0.25	-0.3	-0.25	-0.3	-0.25	0.05	0.2	0.05

		Wind angle with the normal to the front facade							
		0	45	90	135	180	225	270	315
Roof, <10°	Front	-0.5	-0.5	-0.4	-0.5	-0.5	-0.5	-0.4	-0.5
	Rear	-0.5	-0.5	-0.4	-0.5	-0.5	-0.5	-0.4	-0.5
	Mean	-0.5	-0.5	-0.4	-0.5	-0.5	-0.5	-0.4	-0.5
Roof, 11–30°	Front	-0.3	-0.4	-0.5	-0.4	-0.3	-0.4	-0.5	-0.4
	Rear	-0.3	-0.4	-0.5	-0.4	-0.3	-0.4	-0.5	-0.4
	Mean	-0.3	-0.4	-0.5	-0.4	-0.3	-0.4	-0.5	-0.4

The question is: what causes the complaints?

Solution 31 First, the dwelling's steady state heat, air and moisture response during the cold week is analysed. For the air, the dwelling is simplified to a three-node system, one node 1 m above grade representing the first floor, a second 3.75 m above grade representing the second floor and a third 6 m above grade representing the zone below the roof. The heat and vapour balances instead use a single-node approach. Doors between rooms are open daylong, while the floor on grade, the outer walls, and the deck between both floors are considered vapour tight. All calculations use dimensions out to out.

What concerns the airflows between nodes and from outdoors, per node their sum must be zero, or:

$$\sum G_a = 0.$$

Thermal stack in node 1 is 0. In node 2, 2.75 m above node 1, for $\theta_i = 18^\circ\text{C}$ and $\theta_e = -2.5^\circ\text{C}$ it's:

$$p_{T,2} = 2.75 \frac{gP_a}{R_a} \left(\frac{1}{T_i} - \frac{1}{T_e} \right) = -2.48 \text{ Pa}$$

In node 3, 5 m above node 1, for $\theta_i = 18^\circ\text{C}$ and $\theta_{\text{roof}} = -3.9^\circ\text{C}$ due to undercooling, it is

$$p_{T,3} = 5 \frac{gP_a}{R_a} \left(\frac{1}{T_i} - \frac{1}{T_e^*} \right) = -4.84 \text{ Pa}$$

In these formulas, g is the acceleration by gravity (9.81 m/s^2), P_a the atmospheric pressure (some 100 000 Pa) and R_a the gas constant for air ($287 \text{ Pa}\cdot\text{m}^3/(\text{kg}\cdot\text{K})$). The ratio $gP_a/R_a = 3462 \text{ Pa}\cdot\text{K}/\text{m}$.

Stack and wind now give as vertical and horizontal airflows:

$$G_a = a(P_{a,x} + p_{T,x} - P_{a,y} - p_{T,y})^b(1) \quad G_a = a(P_{a,x} - P_{a,y})^b(2)$$

The nodal balances form a system of three equations with the air pressures $P_{a,x}$ unknown:

Node 1

$$a_{e1,1}A_{e1}[P_{a,e1} - P_{a,x1}]^{0.67} + a_{e2,1}A_{e2}[P_{a,e2} - P_{a,x1}]^{0.67} + a_{2,1}A_{2,1}[P_{a,x2} + p_{T,2} - P_{a,x1} - p_{T,1}]^{0.5} = 0$$

Node 2

$$\begin{aligned} & a_{e3,2}A_{e3}[P_{a,e3} - P_{a,x2}]^{0.67} + a_{e4,2}A_{e4}[P_{a,e4} - P_{a,x2}]^{0.67} \\ & + a_{1,2}A_{1,2}[P_{a,x1} + p_{T,1} - P_{a,x2} - p_{T,2}]^{0.5} \\ & + a_{3,2}A_{3,2}[P_{a,x3} + p_{T,3} - P_{a,x2} - p_{T,2}]^{0.5} = 0 \end{aligned}$$

Node 3

$$\begin{aligned} & a_{e5,3}A_{e5}[P_{a,e5} - P_{a,x3}]^{0.67} + a_{e6,3}A_{e6}[P_{a,e6} - P_{a,x3}]^{0.67} \\ & + a_{2,3}A_{2,3}[P_{a,x2} + p_{T,2} - P_{a,x3} - p_{T,3}]^{0.5} = 0 \end{aligned}$$

Solving this system of three equations requires linearization first, followed by a split between known and unknown terms. For that, it is written as:

$$\begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{pmatrix} \cdot \begin{pmatrix} P_{a,x1} \\ P_{a,x3} \\ P_{a,x3} \end{pmatrix} = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix}$$

with the coefficients C_{ij} , and the known terms F_i , equal to

$$\text{Node 1 } C_{11} = - \left(\frac{a_{e1,1}A_{e1}}{\text{abs}(P_{a,e1} - P_{x1})^{0.33}} + \frac{a_{e2,1}A_{e2}}{\text{abs}(P_{a,e2} - P_{x1})^{0.33}} + \frac{a_{2,1}A_{2,1}}{\text{abs}(P_{a,x2} + p_{T,2} - P_{a,x1} - p_{T,1})^{0.5}} \right)$$

$$C_{12} = \frac{a_{2,1}A_{2,1}}{\text{abs}(P_{a,x2} + p_{T,2} - P_{a,x1} - p_{T,1})^{0.5}}$$

$$C_{13} = 0$$

$$F_1 = - \left(\frac{a_{e1,1}A_{e1}P_{a,e1}}{\text{abs}(P_{a,e1} - P_{x1})^{0.33}} + \frac{a_{e2,1}A_{e2}P_{a,e2}}{\text{abs}(P_{a,e2} - P_{x1})^{0.33}} + \frac{a_{2,1}A_{2,1}(p_{T,2} - p_{T,1})}{\text{abs}(P_{a,x2} + p_{T,2} - P_{a,x1} - p_{T,1})^{0.5}} \right)$$

$$\text{Node 2 } C_{21} = \frac{a_{1,2}A_{1,2}}{\text{abs}(P_{a,x1} + p_{T,1} - P_{a,x2} - p_{T,2})^{0.5}}$$

$$C_{22} = - \left(\frac{a_{e3,2}A_{e3}}{\text{abs}(P_{a,e3} - P_{x2})^{0.33}} + \frac{a_{e4,2}A_{e4}}{\text{abs}(P_{a,e4} - P_{x2})^{0.33}} + \frac{a_{1,2}A_{1,2}}{\text{abs}(P_{a,x1} + p_{T,1} - P_{a,x2} - p_{T,2})^{0.5}} + \frac{a_{3,2}A_{3,2}}{\text{abs}(P_{a,x3} + p_{T,3} - P_{a,x2} - p_{T,2})^{0.5}} \right)$$

$$C_{23} = \frac{a_{3,2}A_{3,2}}{\text{abs}(P_{a,x3} + p_{T,3} - P_{a,x2} - p_{T,2})^{0.5}}$$

$$F_2 = - \left(\begin{aligned} & \frac{a_{e3,2} A_{e2} P_{a,e3}}{\text{abs}(P_{a,e3} - P_{x2})^{0.33}} + \frac{a_{e2,1} A_{e2} P_{a,e4}}{\text{abs}(P_{a,e4} - P_{x2})^{0.33}} \\ & + \frac{a_{1,2} A_{1,2} (p_{T,1} - p_{T,2})}{\text{abs}(P_{a,x1} + p_{T,1} - P_{a,x2} - p_{T,2})^{0.5}} \\ & + \frac{a_{3,2} A_{3,2} (p_{T,3} - p_{T,2})}{\text{abs}(P_{a,x3} + p_{T,2} - P_{a,x2} - p_{T,3})^{0.5}} \end{aligned} \right)$$

Node 3 $C_{31} = 0$

$$C_{32} = - \left(\begin{aligned} & \frac{a_{e5,3} A_{e5}}{\text{abs}(P_{a,e5} - P_{x3})^{0.33}} + \frac{a_{e6,3} A_{e6}}{\text{abs}(P_{a,e6} - P_{x3})^{0.33}} \\ & + \frac{a_{2,3} A_{2,3}}{\text{abs}(P_{a,x2} + p_{T,2} - P_{a,x3} - p_{T,3})^{0.5}} \end{aligned} \right)$$

$$C_{33} = \left(\frac{a_{2,3} A_{2,3}}{\text{abs}(P_{a,x2} + p_{T,2} - P_{a,x3} - p_{T,3})^{0.5}} \right)$$

$$F_3 = - \left(\begin{aligned} & \frac{a_{e5,3} A_{e5} P_{a,e5}}{\text{abs}(P_{a,e5} - P_{x3})^{0.33}} + \frac{a_{e6,3} A_{e6} P_{a,e6}}{\text{abs}(P_{a,e6} - P_{x3})^{0.33}} \\ & + \frac{a_{2,3} A_{2,3} (p_{T,2} - p_{T,3})}{\text{abs}(P_{a,x2} + p_{T,2} - P_{a,x3} - p_{T,3})^{0.5}} \end{aligned} \right)$$

To solve, the known pressures and temperatures are inserted together with guessed values for the three unknown pressures $P_{a,x1}$, $P_{a,x2}$ and $P_{a,x3}$. This allows calculating the coefficients C_{ij} and the known terms F_i . After solving, the C_{ij} 's and F_i 's are recalculated with the three-node pressures found and the system is resolved. Iterating so goes on till the root of the summed quadratic deviations between new and previous drops below a pre-set value: $a_{\text{preset}} < \sqrt{\sum \varepsilon^2}$

With $n_{50} = 10$ ach, quantifying the air permeance of the façade at the front and the rear starts with calculating the roof's mean air permeance coefficient:

$$K_{a,\text{roof}} = 3.3 \cdot 10^{-4} / 1.2 = 2.75 \cdot 10^{-4} \text{ m}^3 / (\text{m}^2 \cdot \text{s} \cdot \text{Pa}^b)$$

A ventilation rate 10 ach at 50 Pa for a net air volume of 248.3 m^3 means an airflow equal to $(10.4 \cdot 248.3 / 3600) / 50^{0.67} = 0.0502 \text{ m}^3 / \text{s}$ at 1 Pa, of which $(3.3 \cdot 10^{-4} / 1.2) \cdot 54.7 = 0.0154 \text{ m}^3 / \text{s}$ passes the roof, leaving $0.037 \text{ m}^3 / \text{s}$ for the front and rear. Assuming each half of both façades shows an equal leakage, the result is $0.037 / 4 = 9.25 \cdot 10^{-3} \text{ m}^3 / (\text{s} \cdot \text{Pa}^b)$.

The flow equations for the open staircase and the link between node 2 and 3 follows from conservation of energy, stating that the difference in pressure and stack should equal the change in kinetic energy of the airflow. This gives as permeance coefficients:

$$\text{Staircase: } 4.0 \text{ m}^3 / (\text{s} \cdot \text{Pa}^{0.5}) \quad \text{First floor to roof zone: } 56.2 \text{ m}^3 / (\text{s} \cdot \text{Pa}^{0.5})$$

The stack pressures are already calculated. When the wind blows NE and the front façade looks NE, those the wind induces become: